Auto Encoders

What PCA wished it was

Introduction to Machine Learning, Spring 2026

Recall: Motivation for PCA

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Lower dimension $\rightarrow$ computational efficiency, lower redundancy (list goes on)

What can go wrong?

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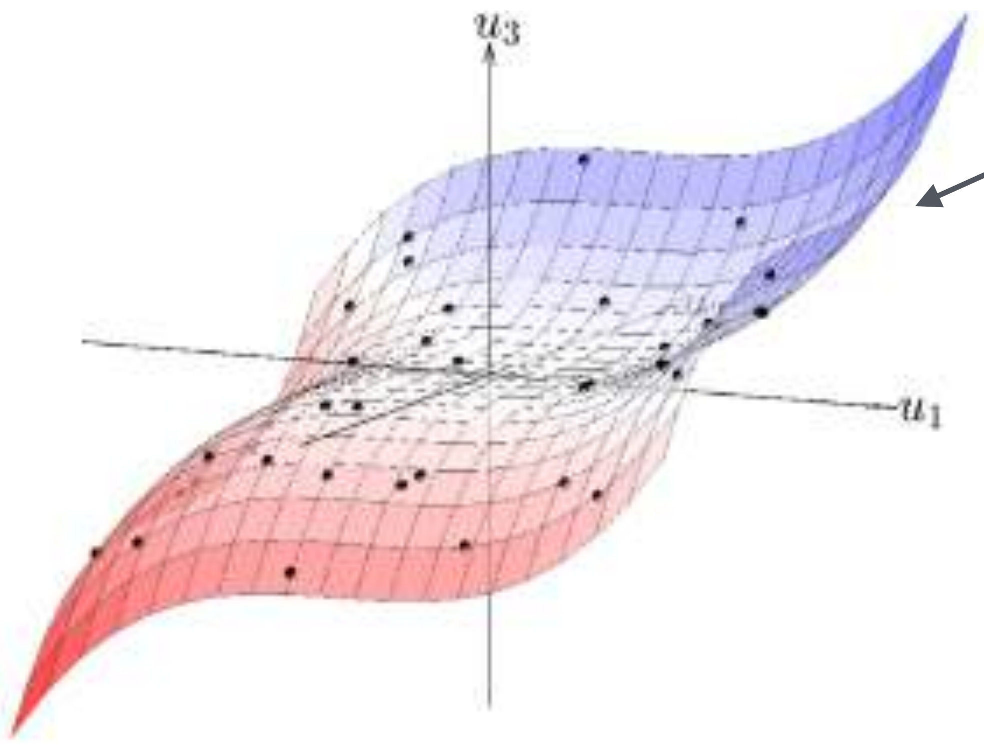
Assumes that data lies on a smaller dimensional *linear* manifold



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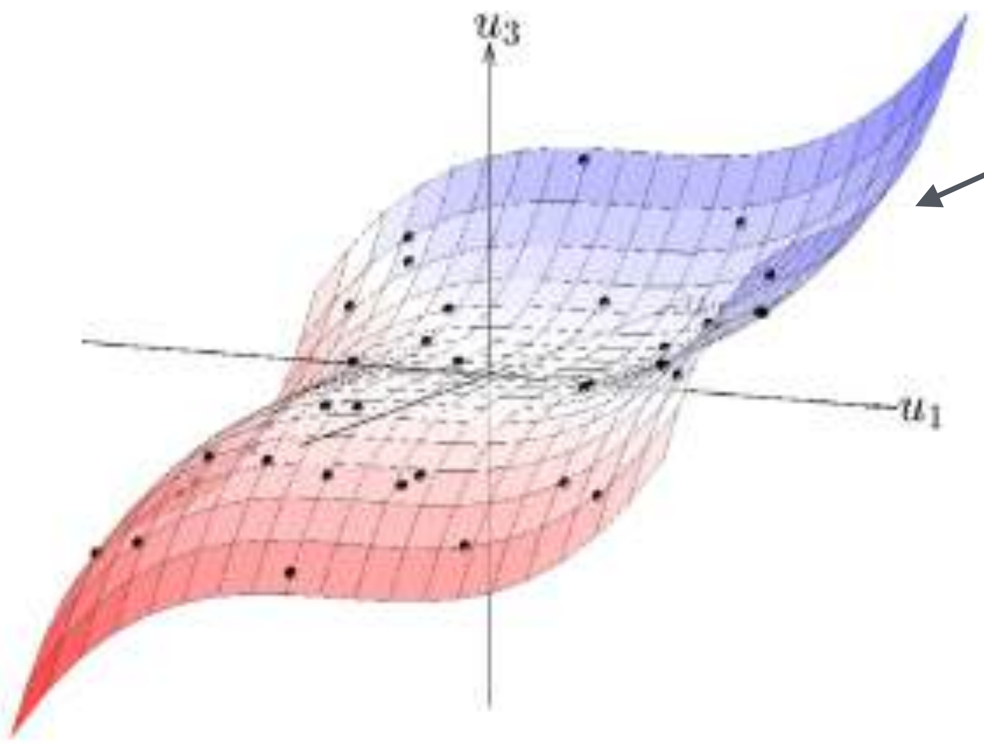

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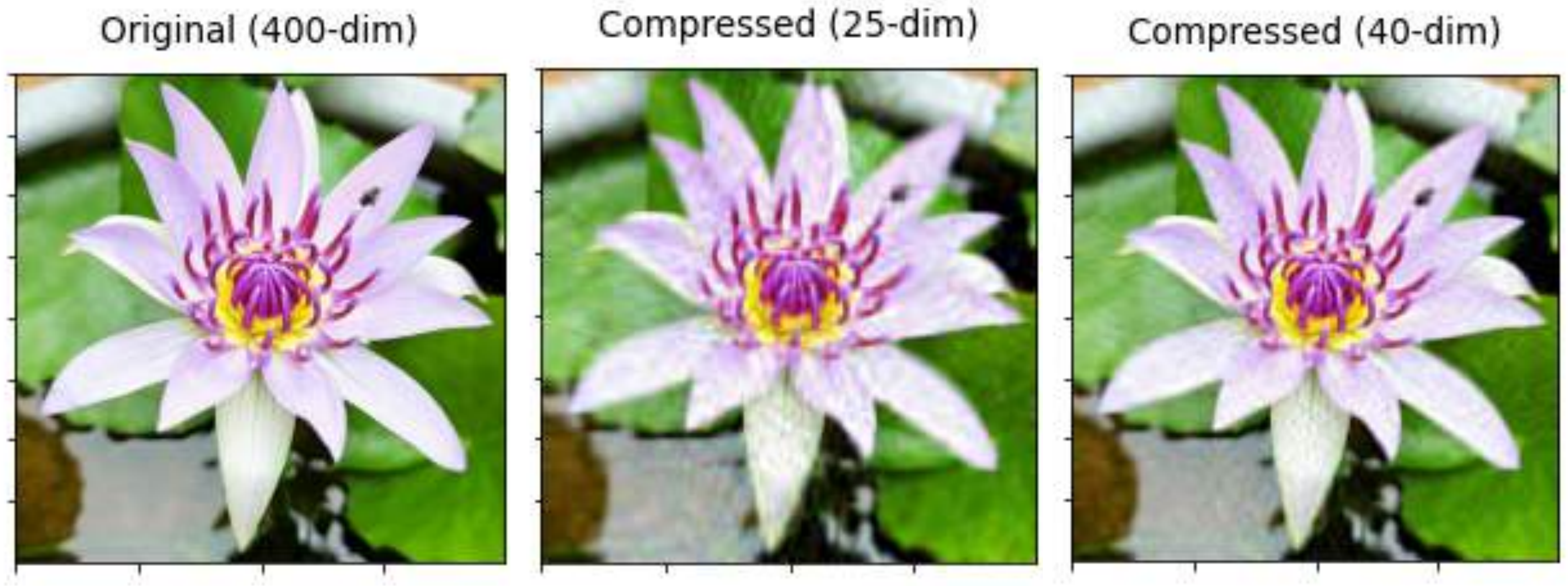

It is lossy - the lower dimensional representations cannot be used to get the original data back.

Can we *learn*?

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We need a clever way to convert this unsupervised learning task into a supervised learning task

A random problem

A random problem

Let's consider a completely different problem

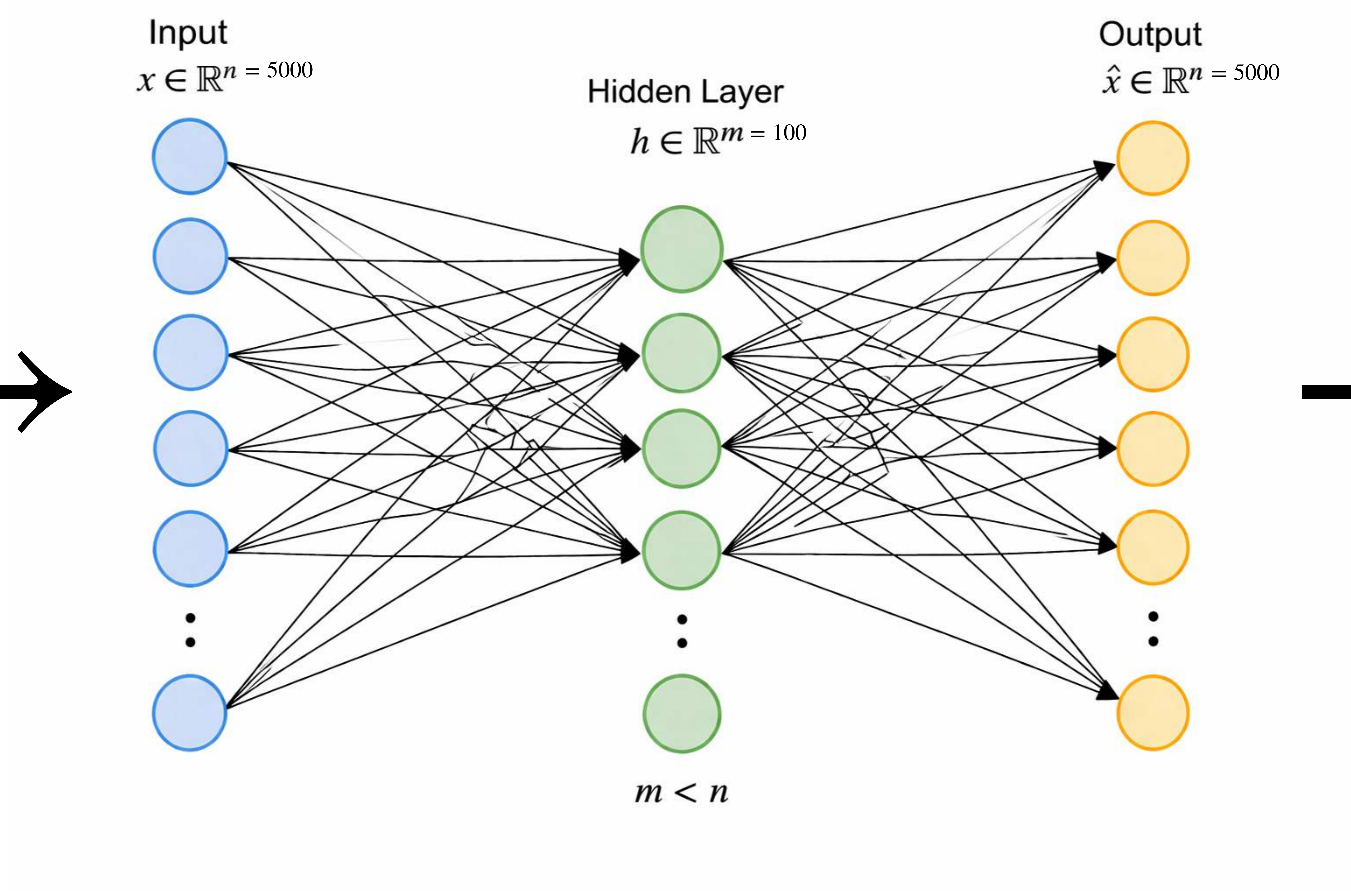
A random problem

Let's consider a completely different problem

Suppose I train a network, such that given x, it outputs.. x

A random problem

$x \rightarrow$



$\rightarrow x$

A random problem

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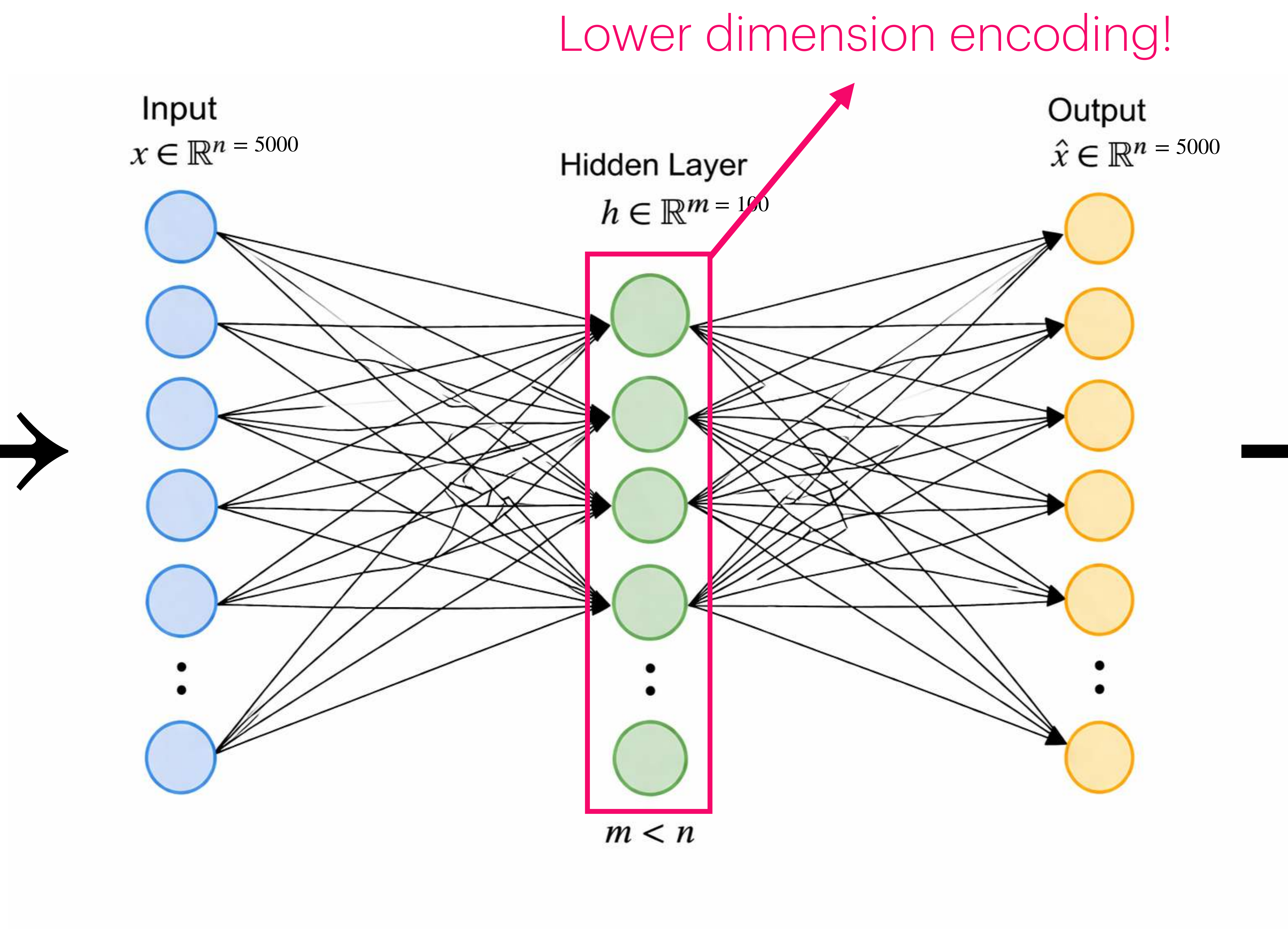

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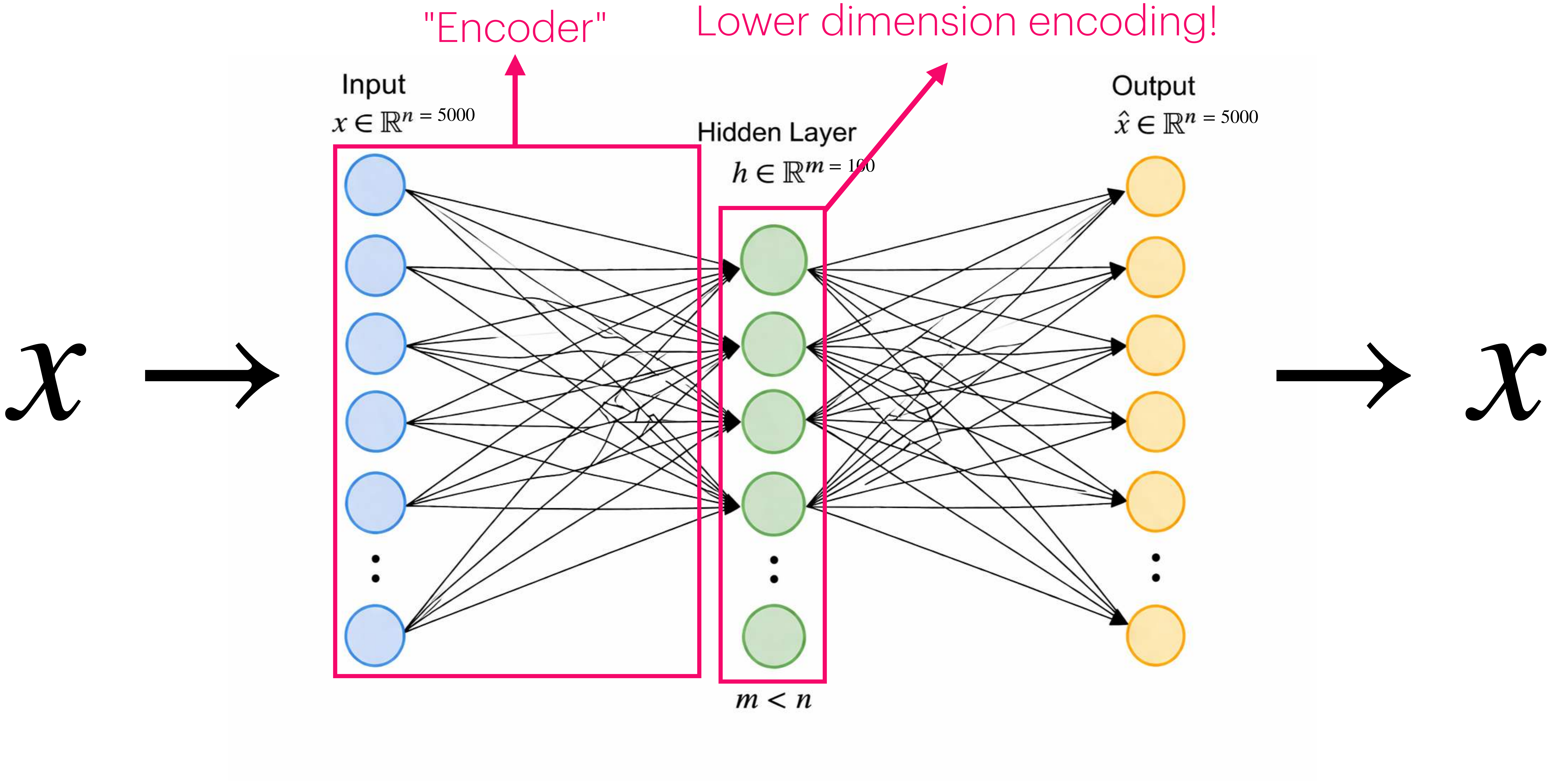

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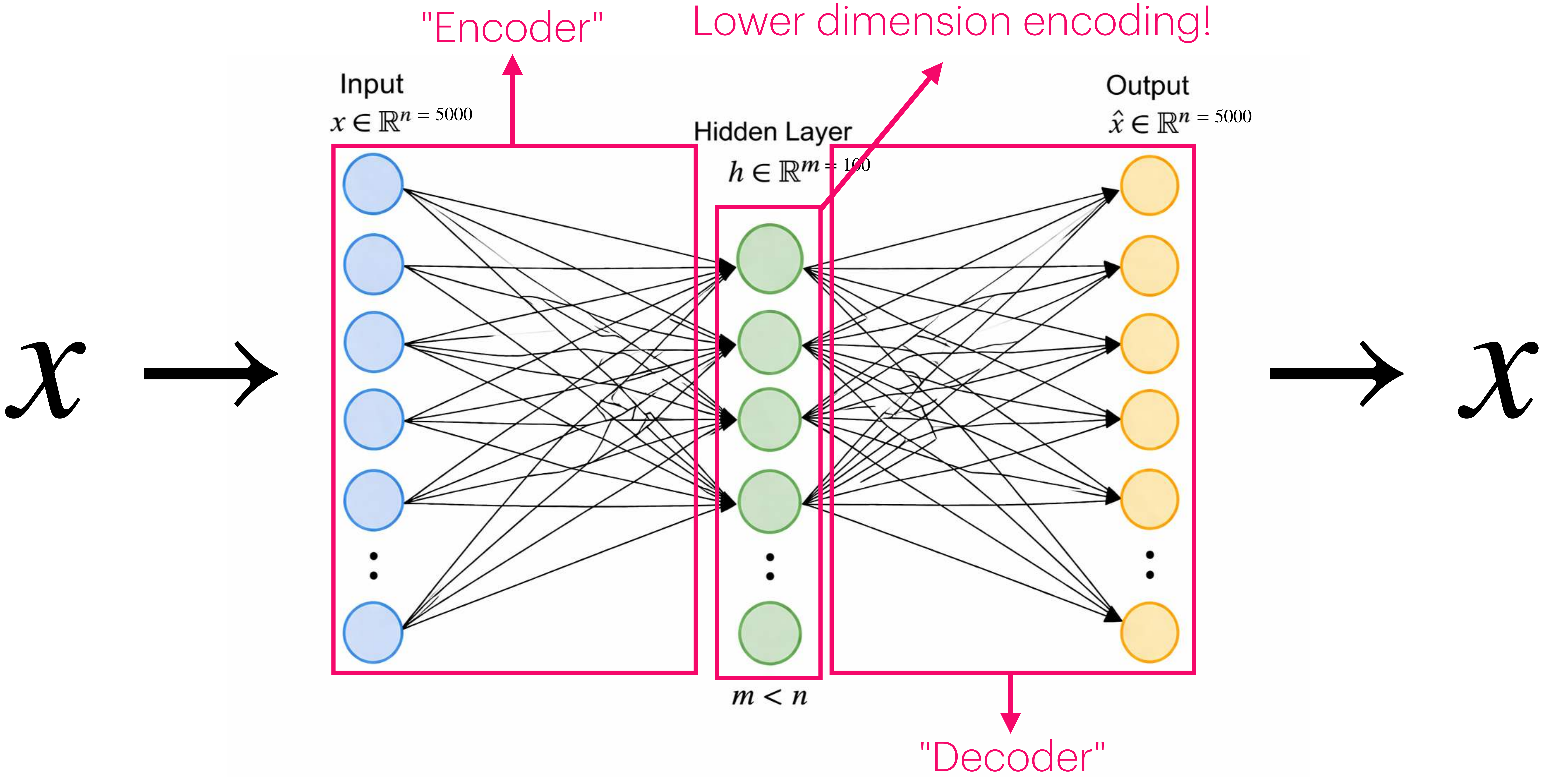

Simplifying the problem

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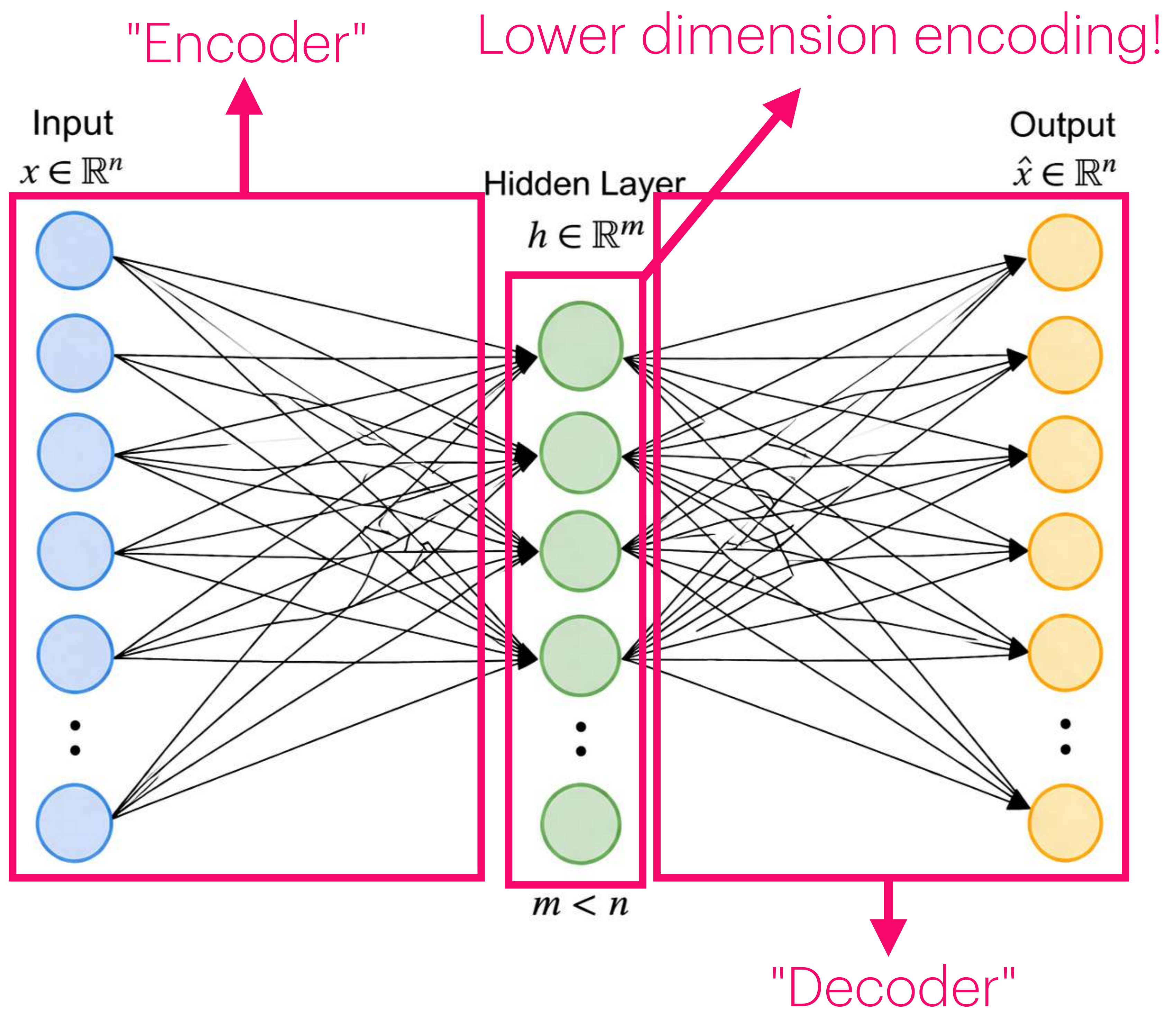

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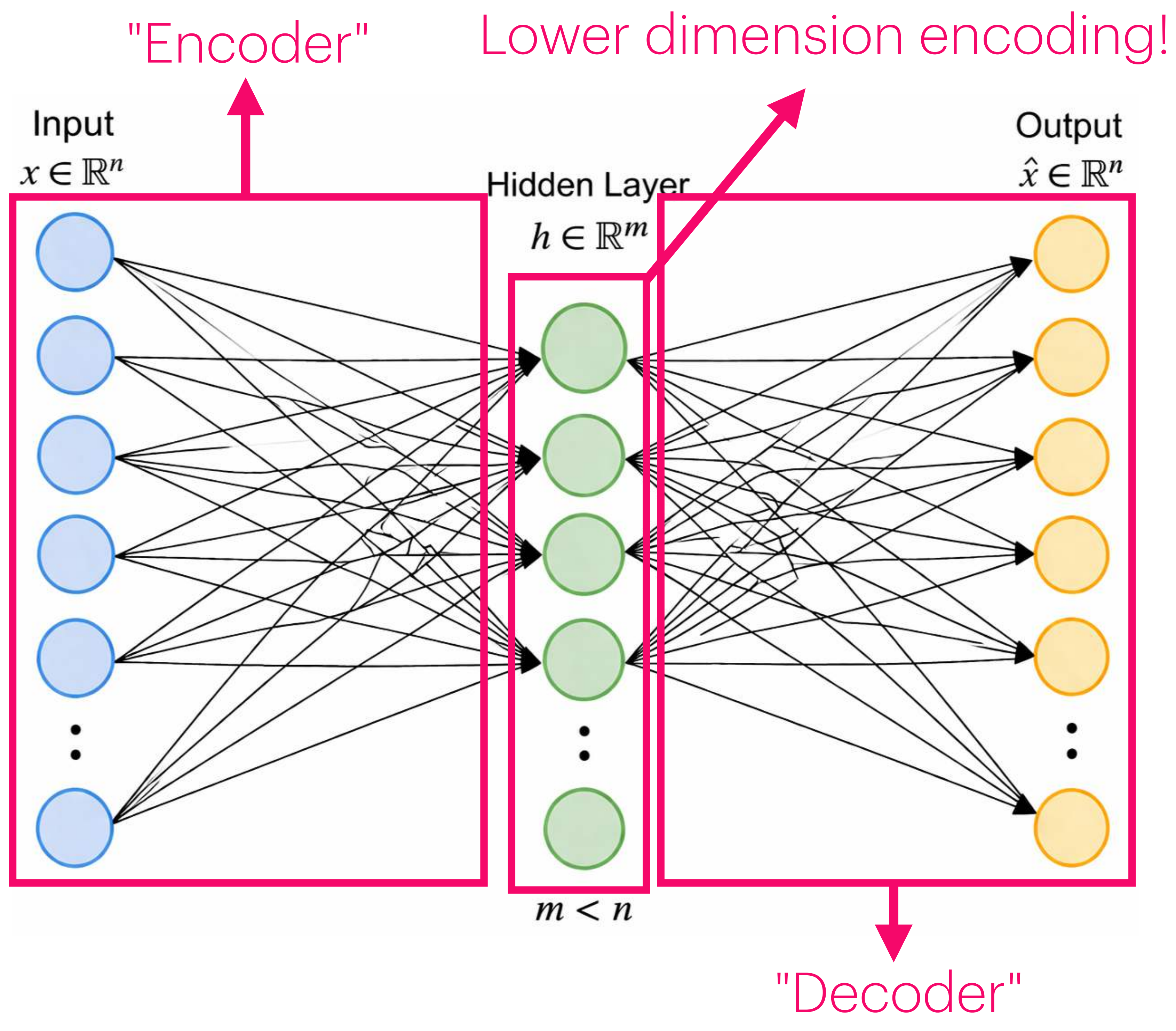

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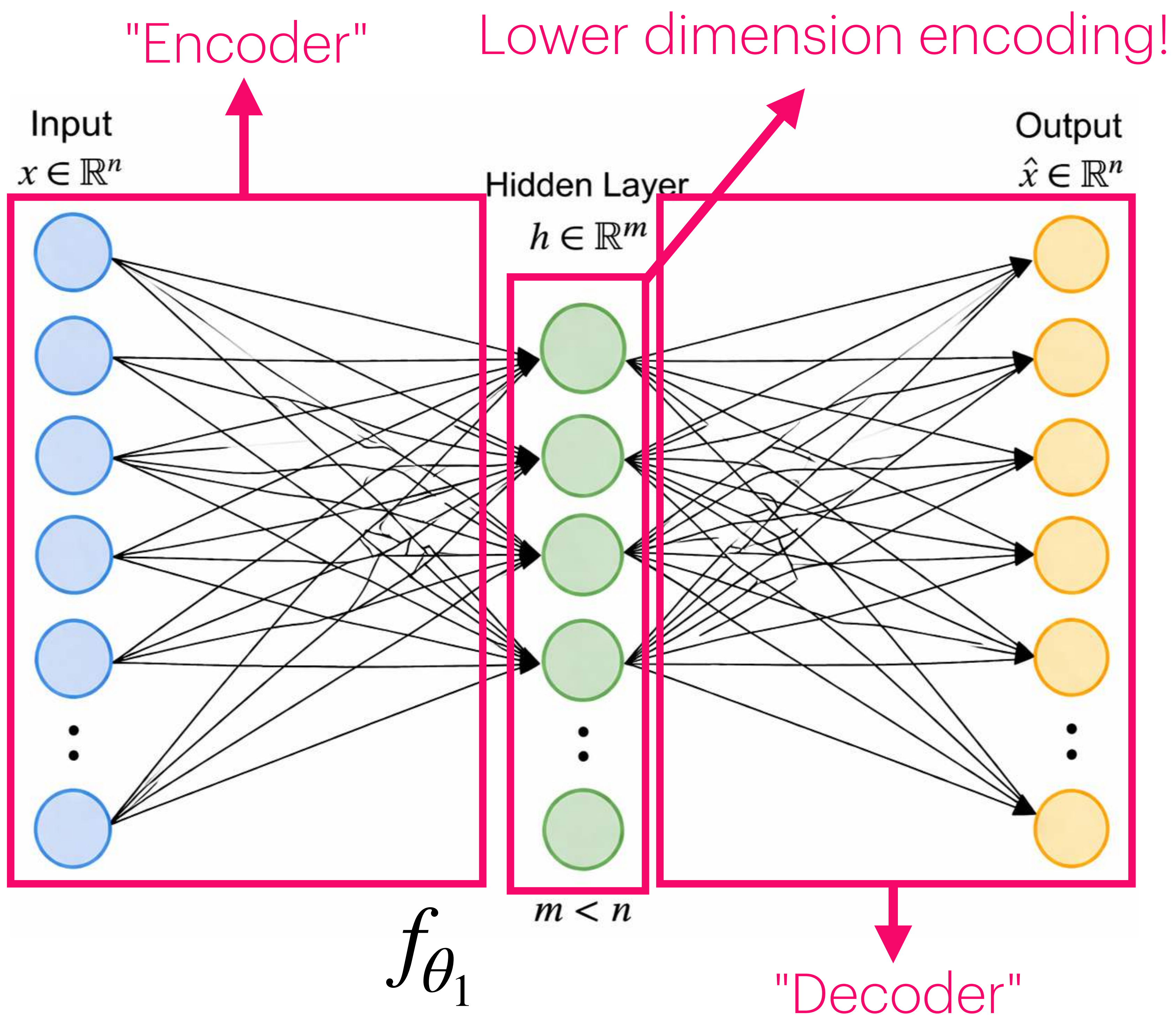
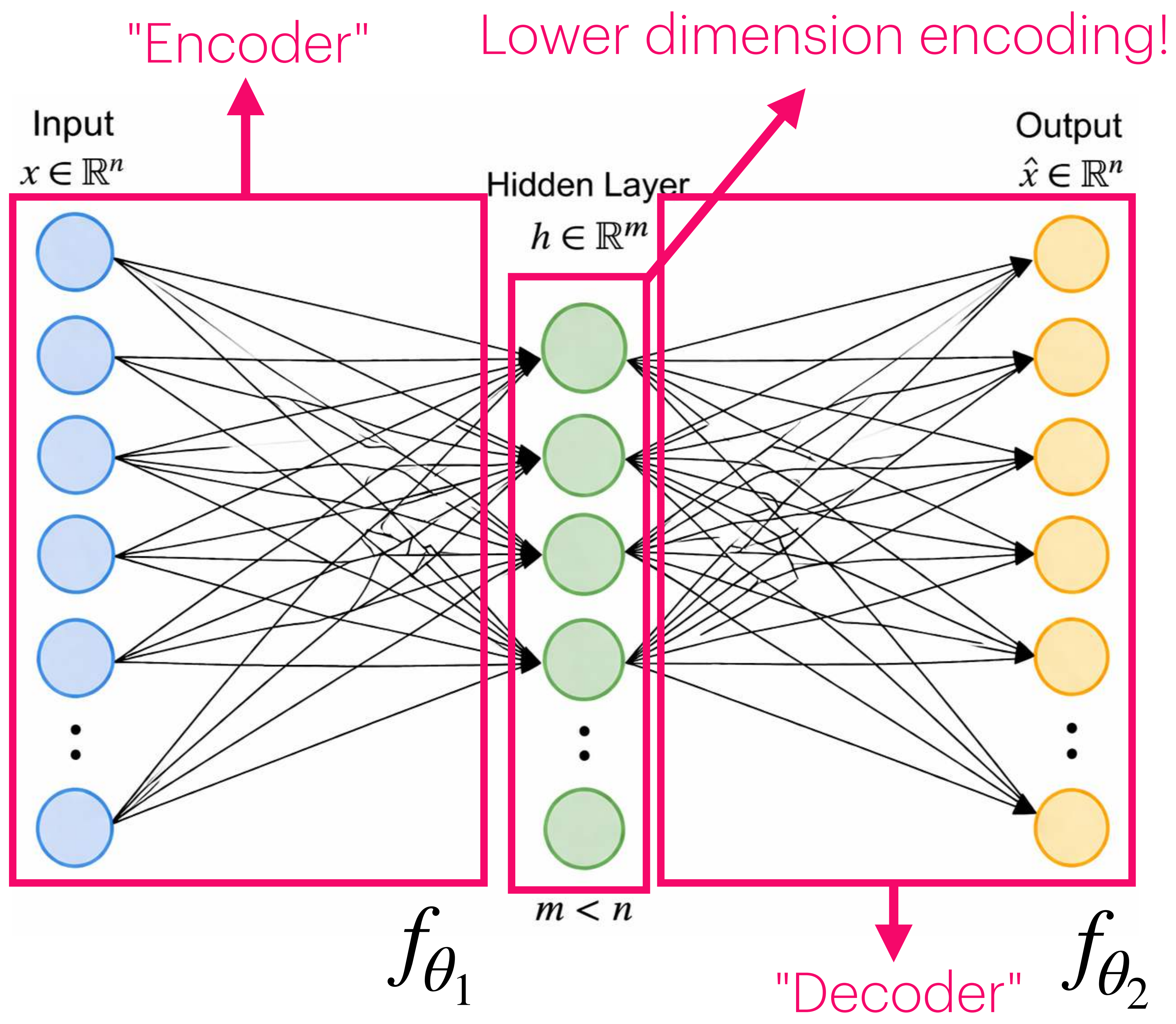


$\rightarrow$

$f_{\theta_1} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ – the encoder

Simplifying the problem



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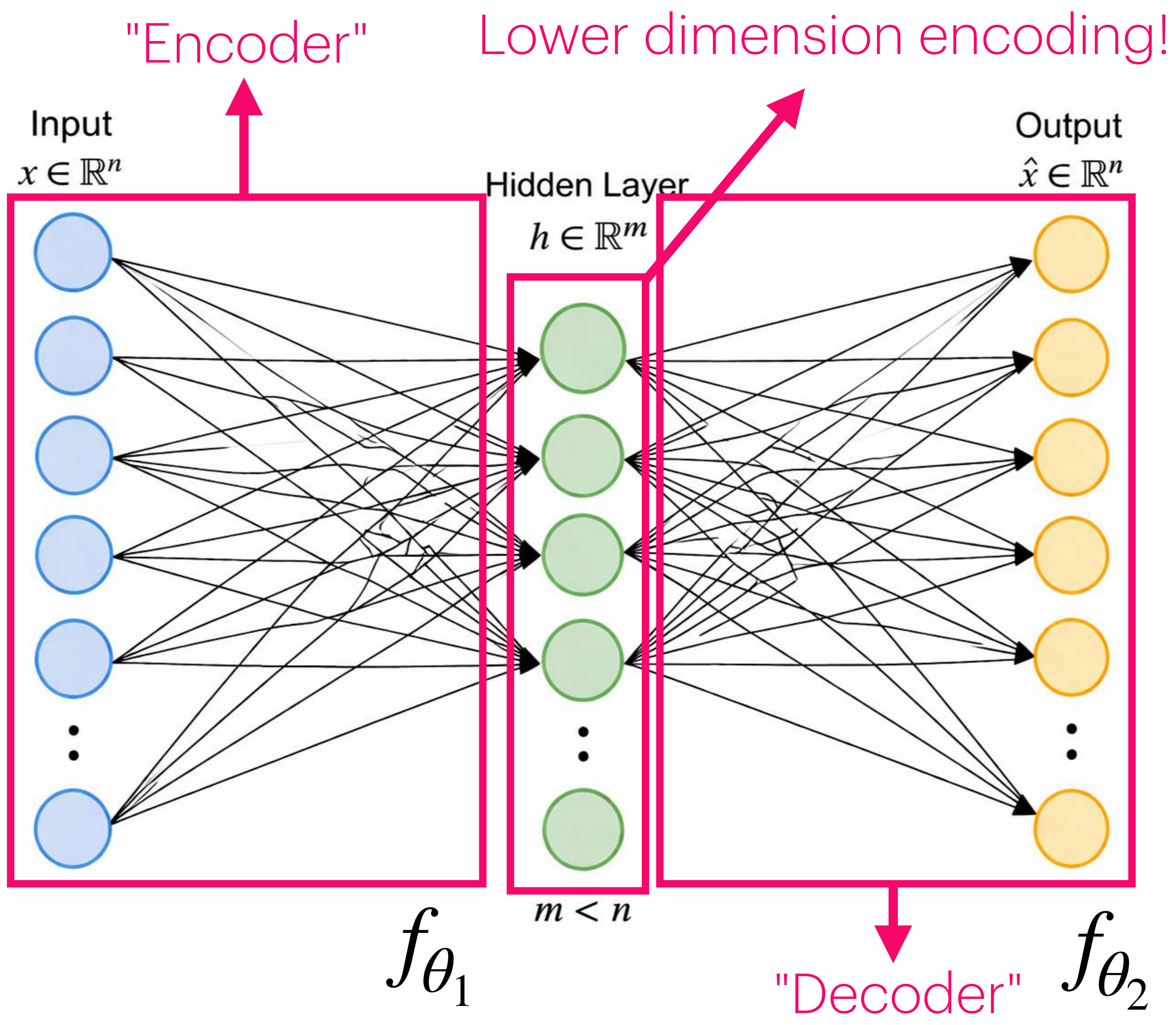

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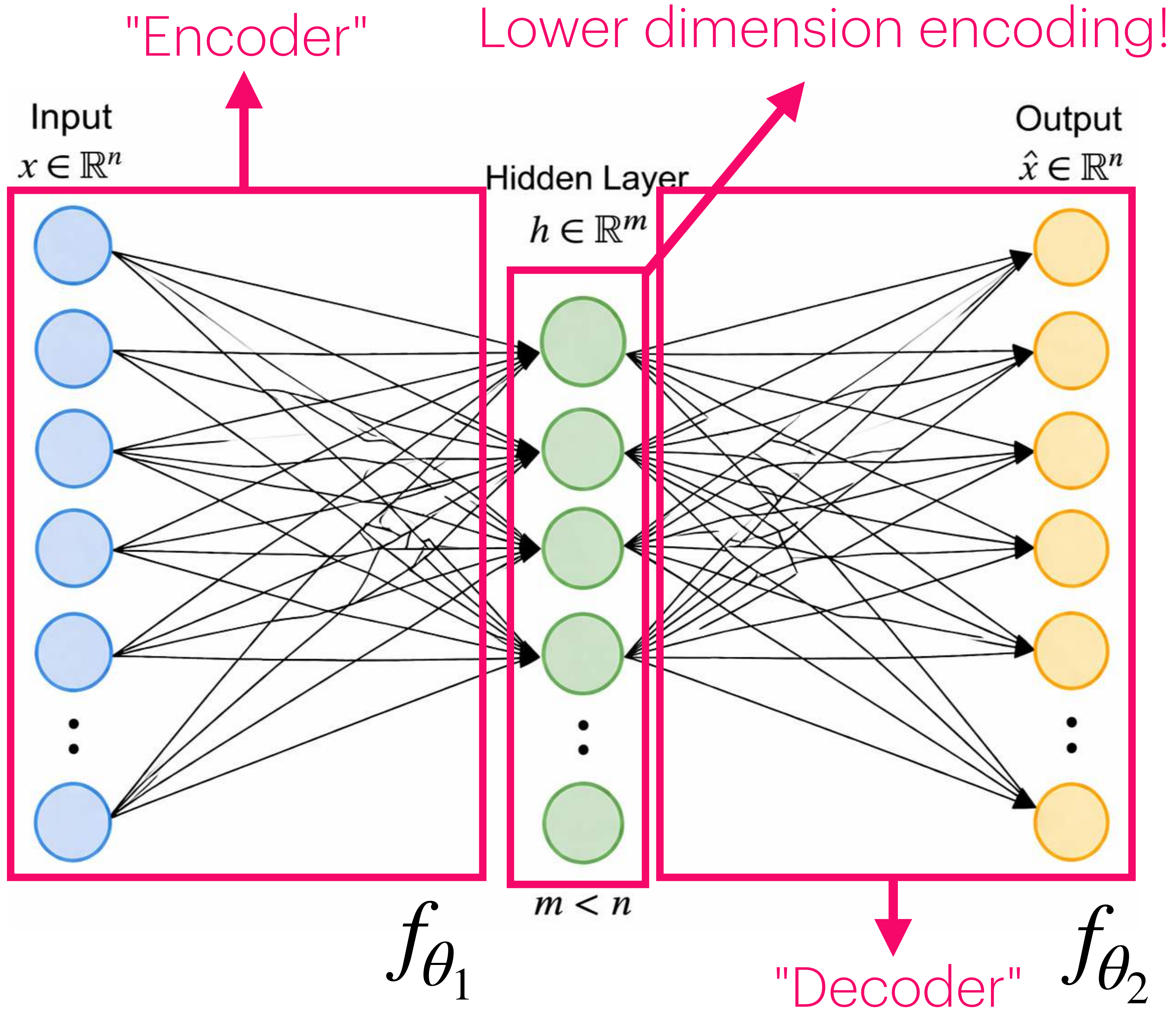

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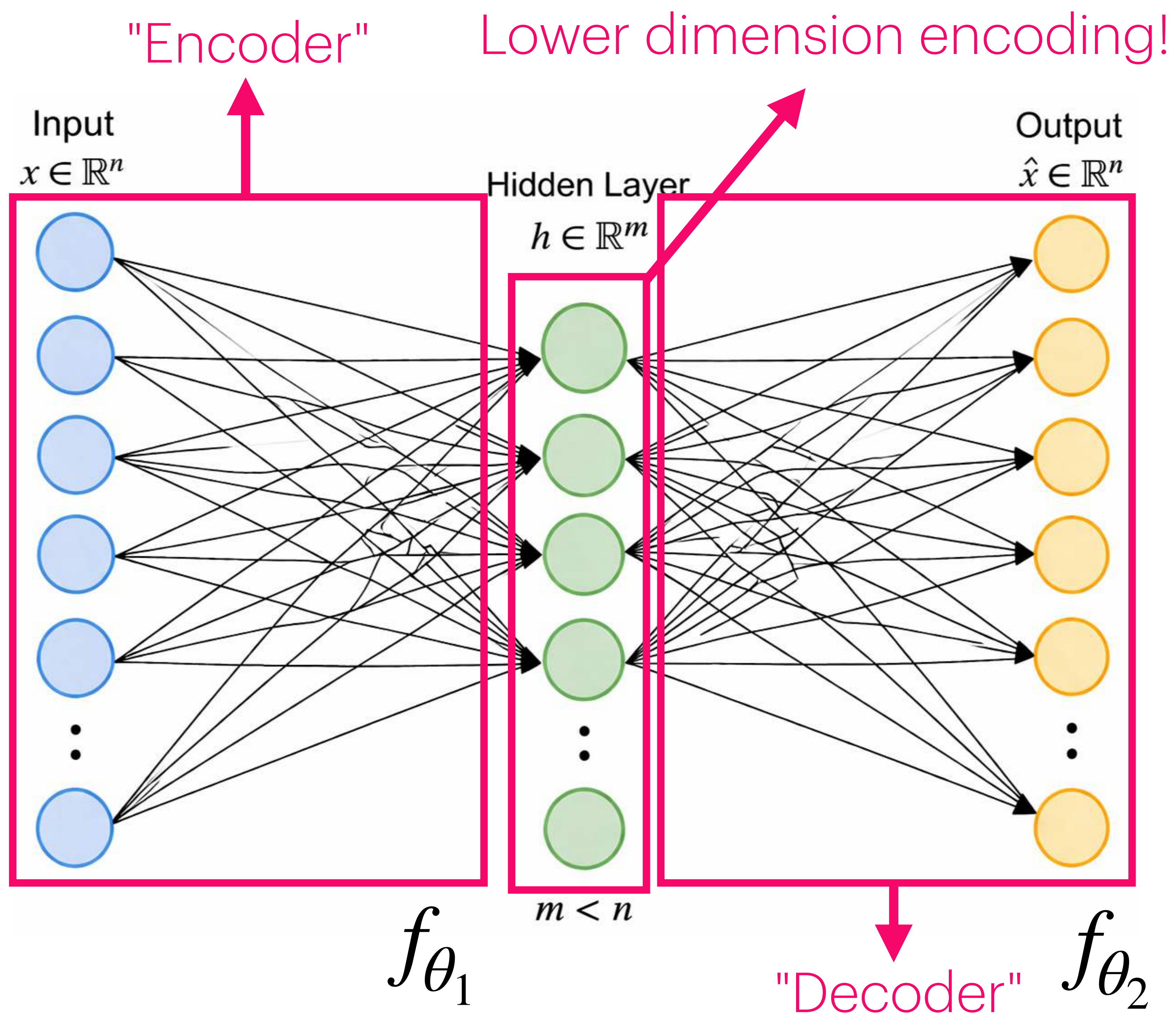

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$$f_{\theta_2}(f_{\theta_1}(x)) = x$$

The architecture

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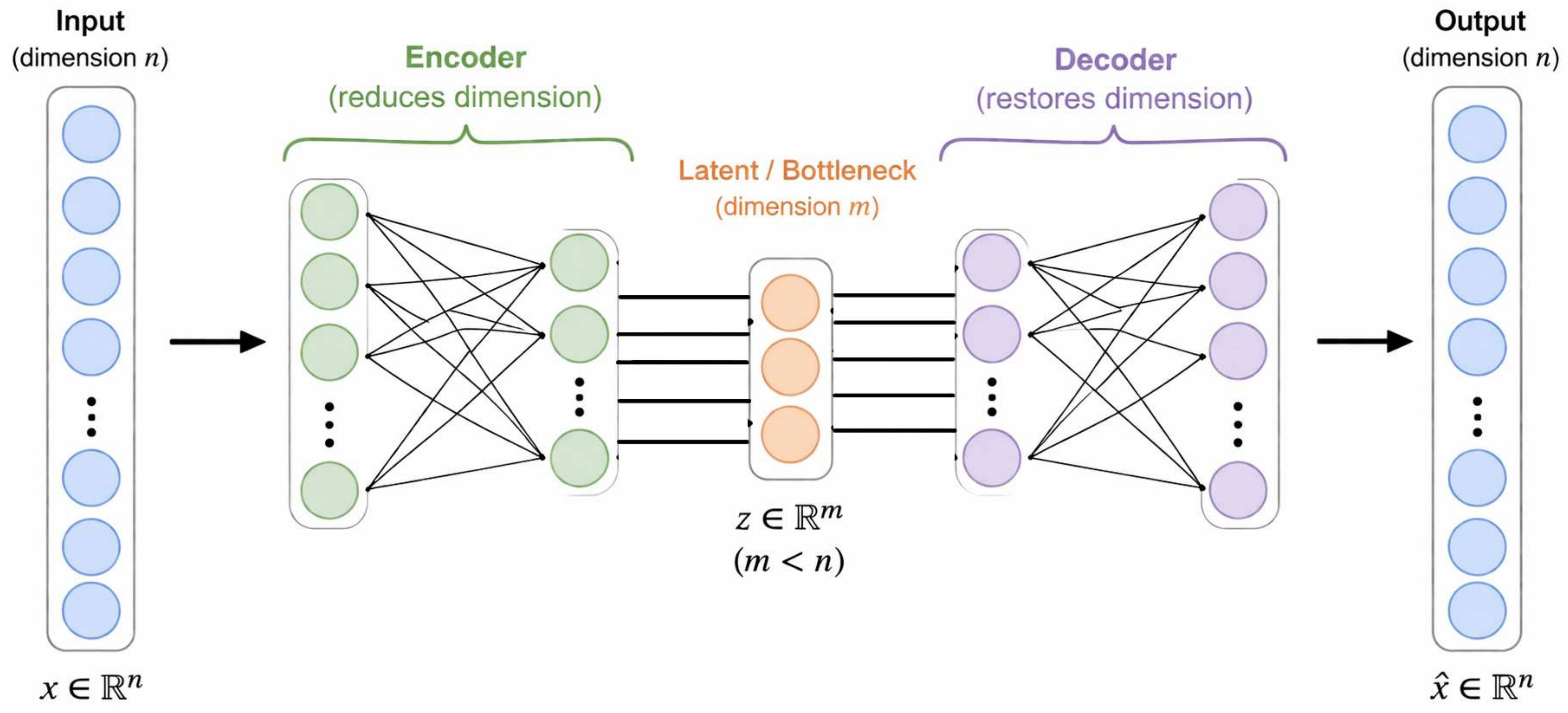

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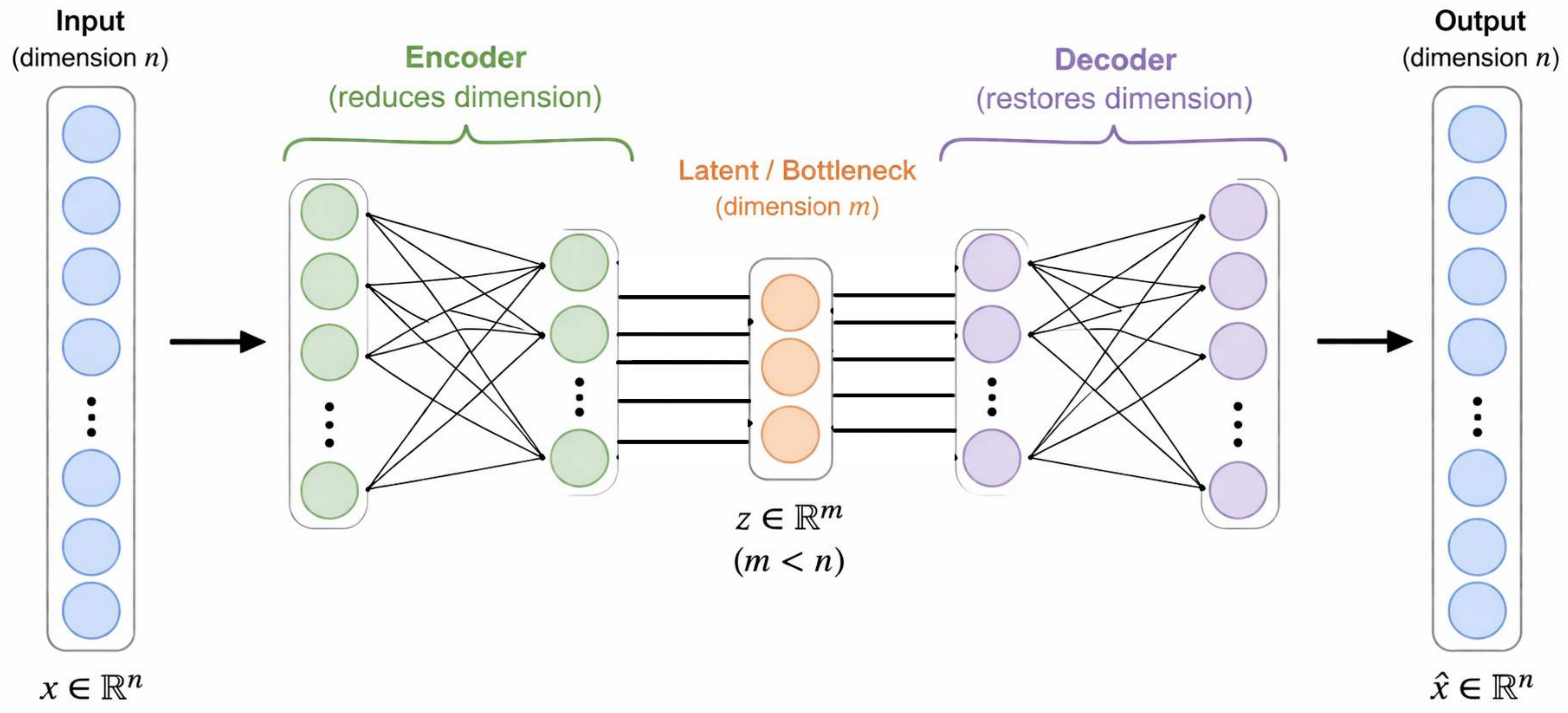
$x \rightarrow$



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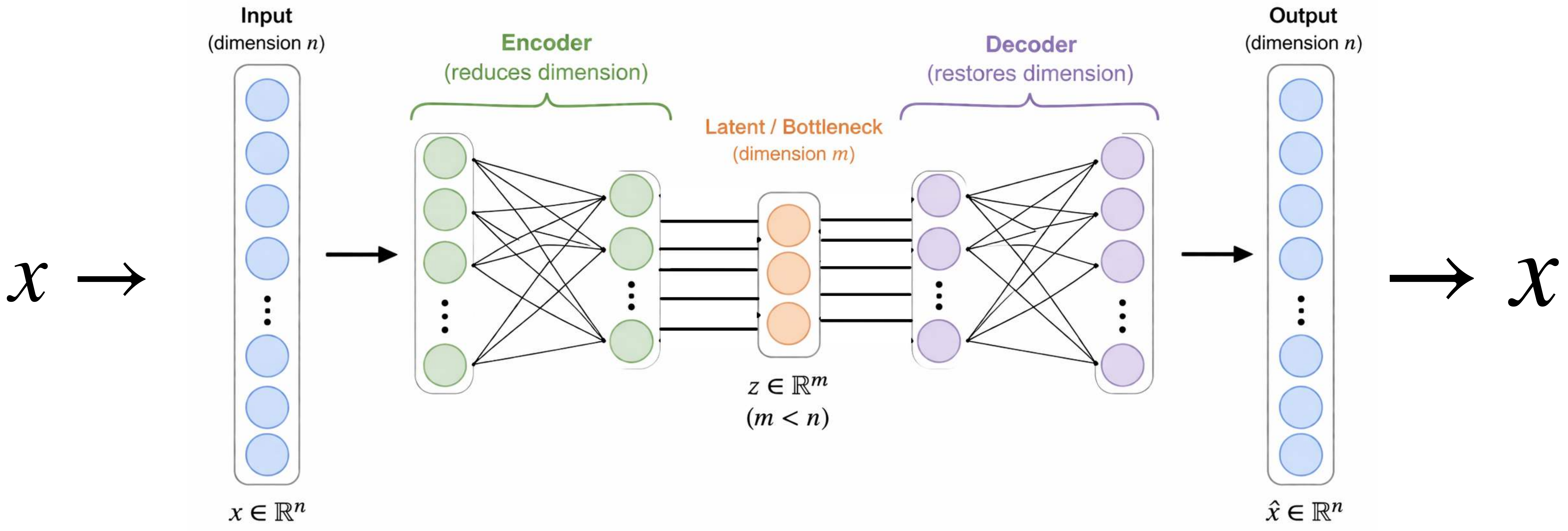

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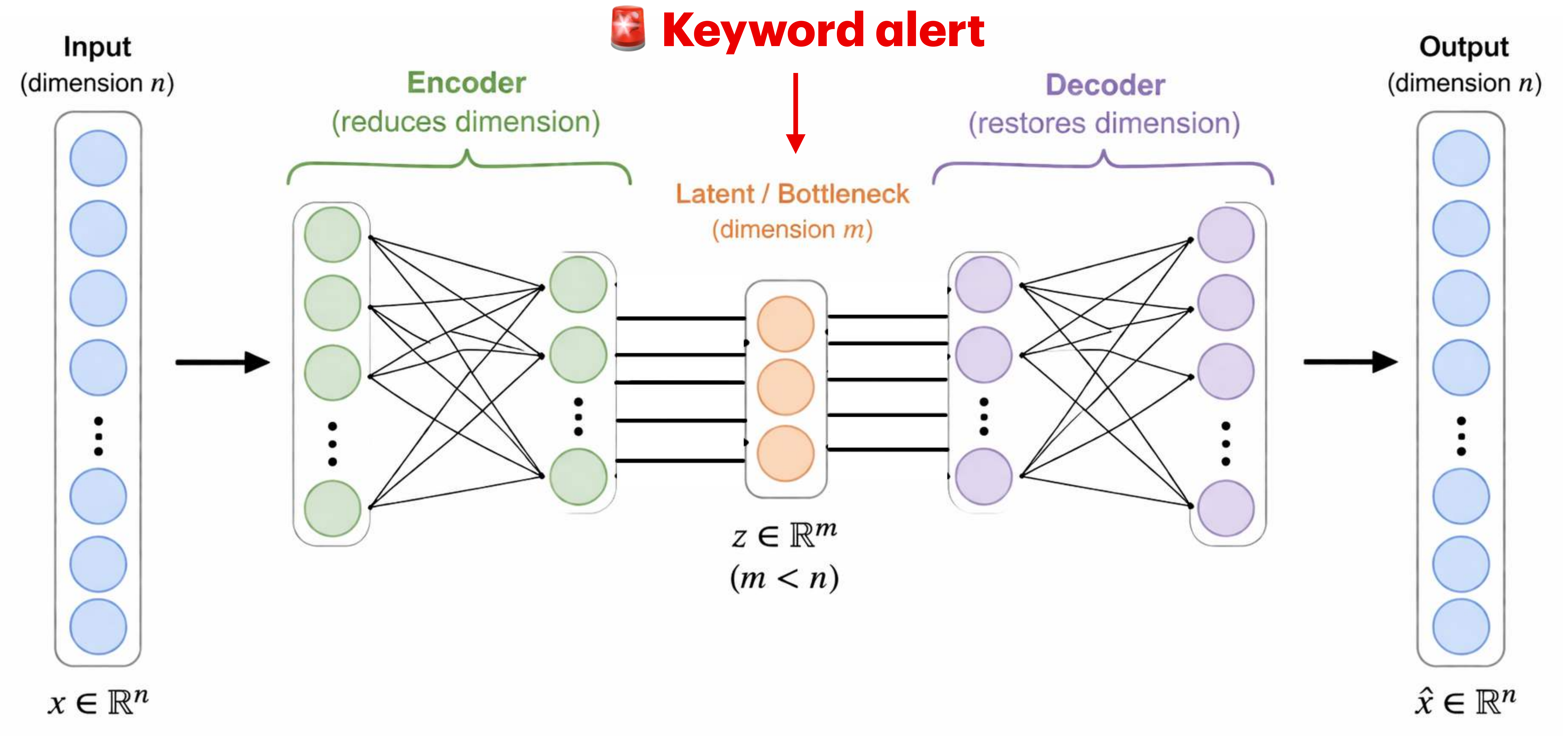

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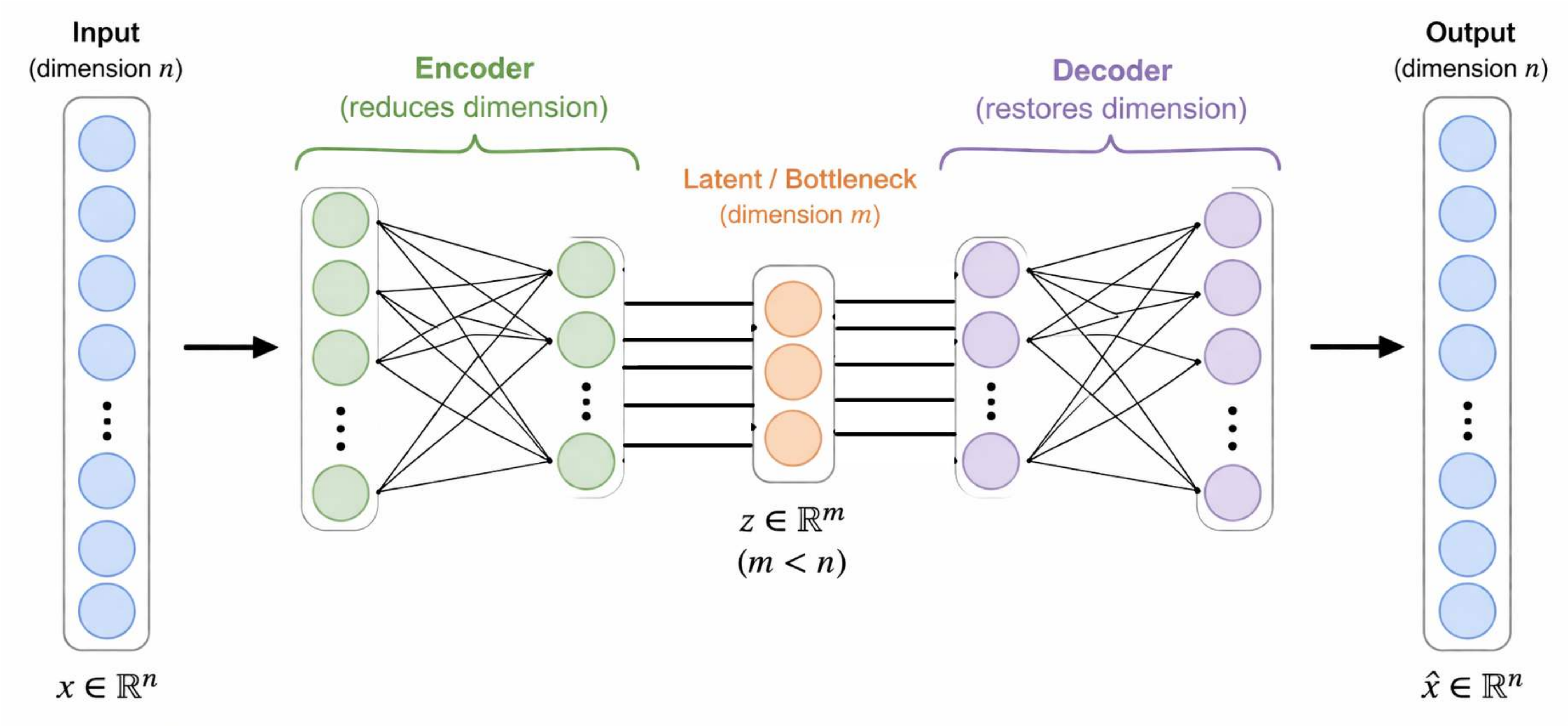

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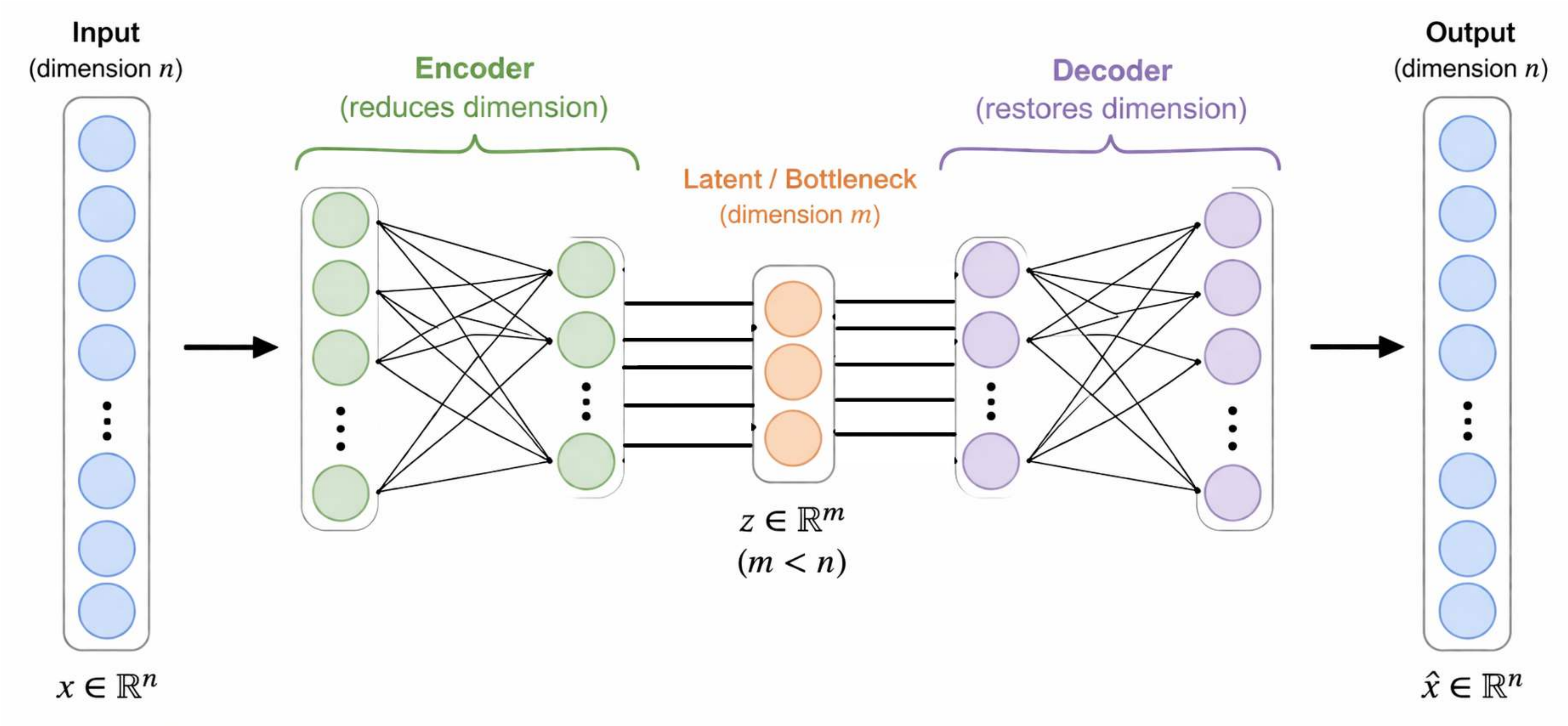
Hyperparameters?



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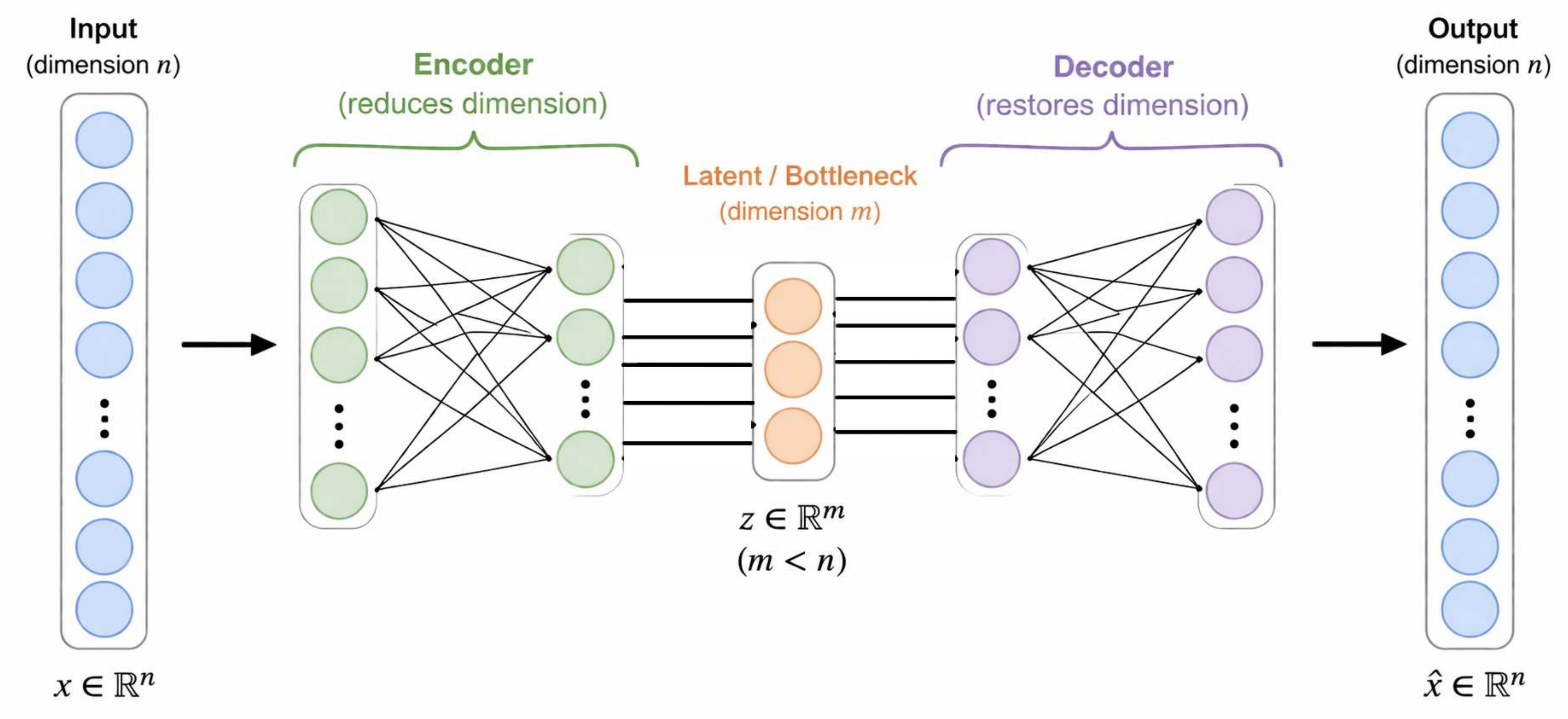

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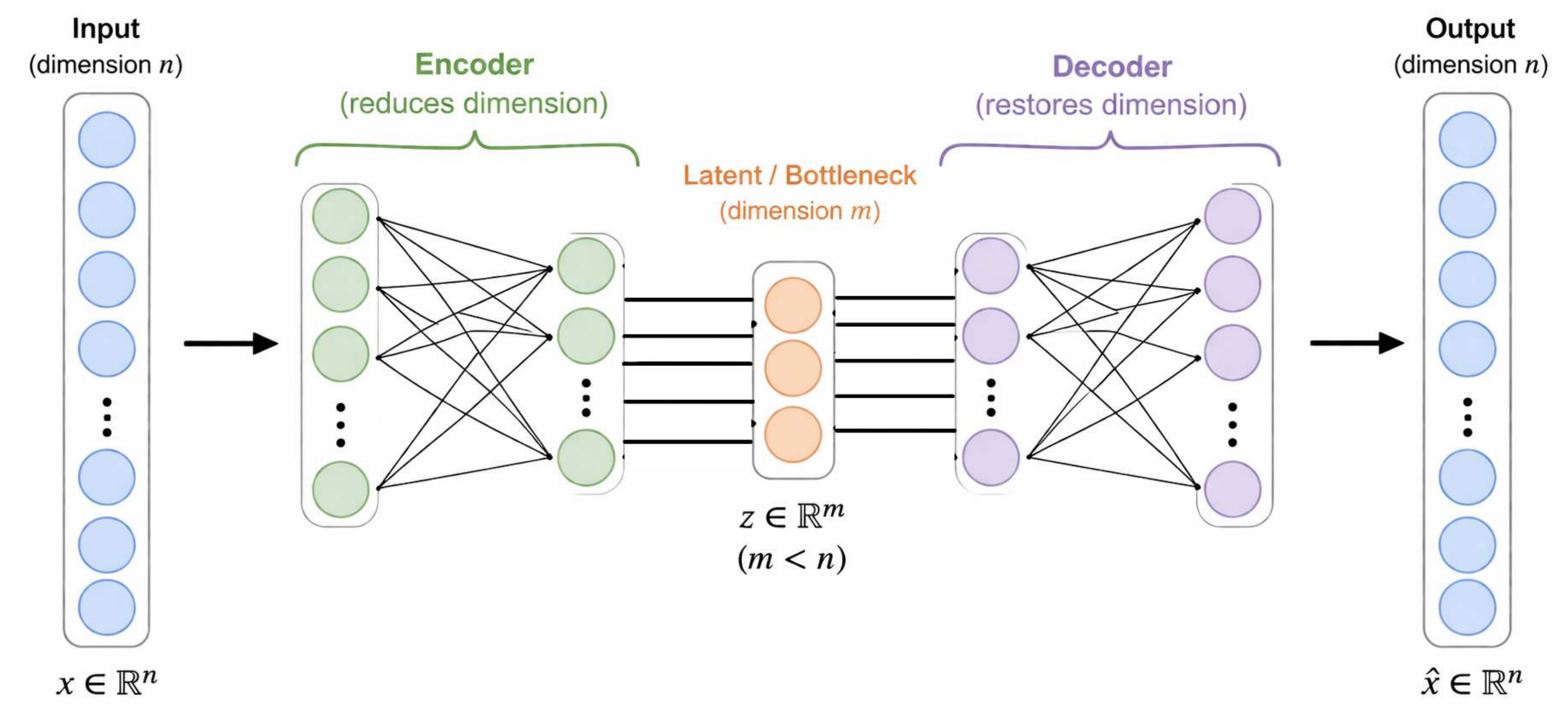

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Compression quality?

Expressiveness of neural network. Size of latent dimension. Training duration

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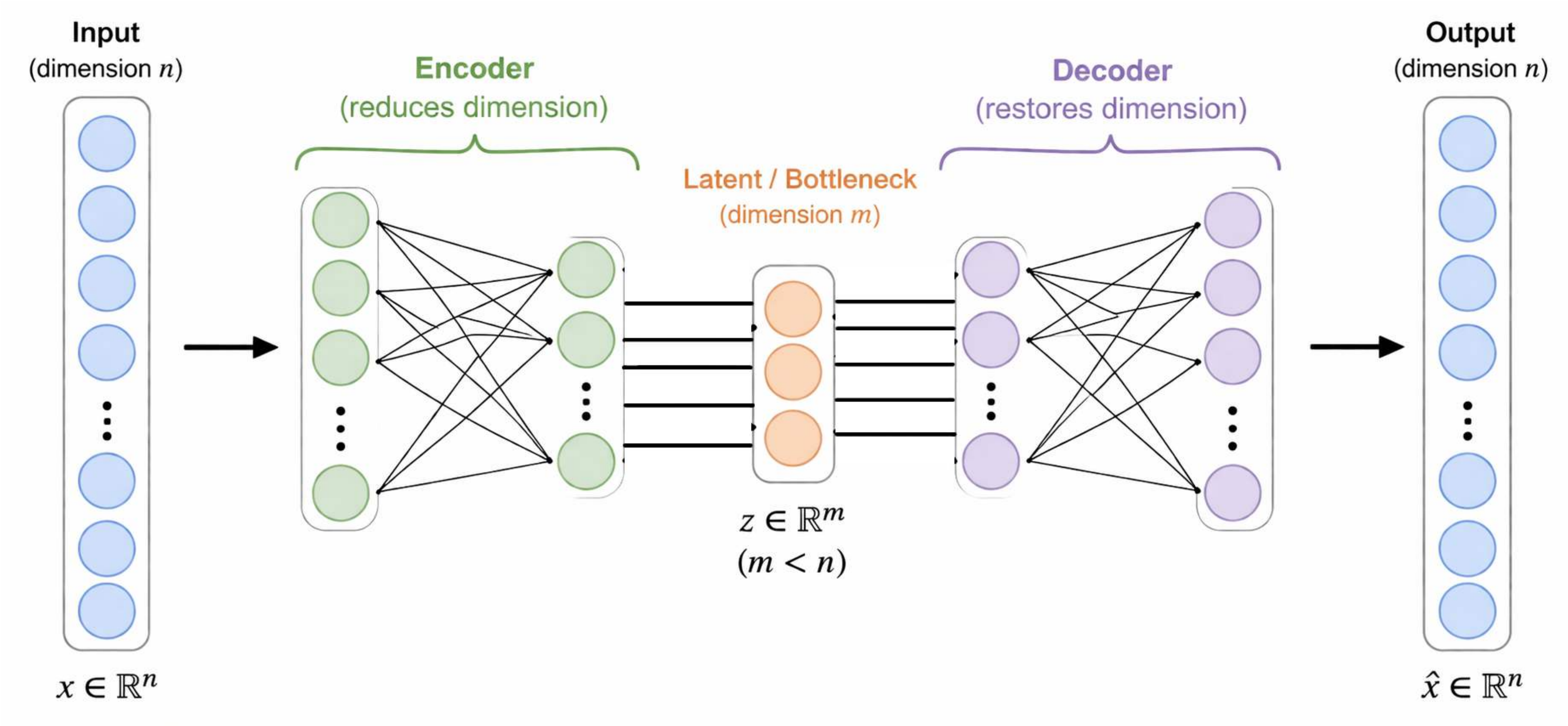

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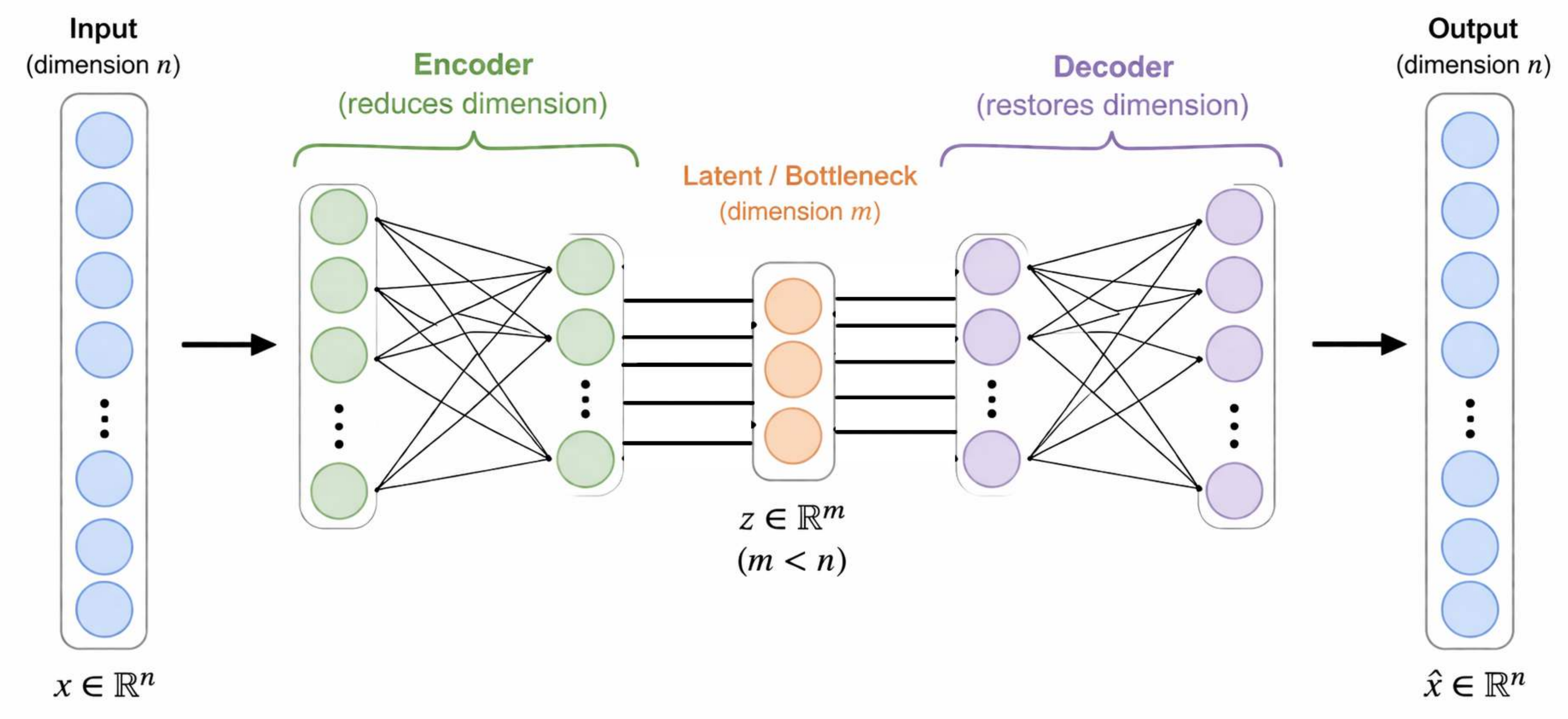

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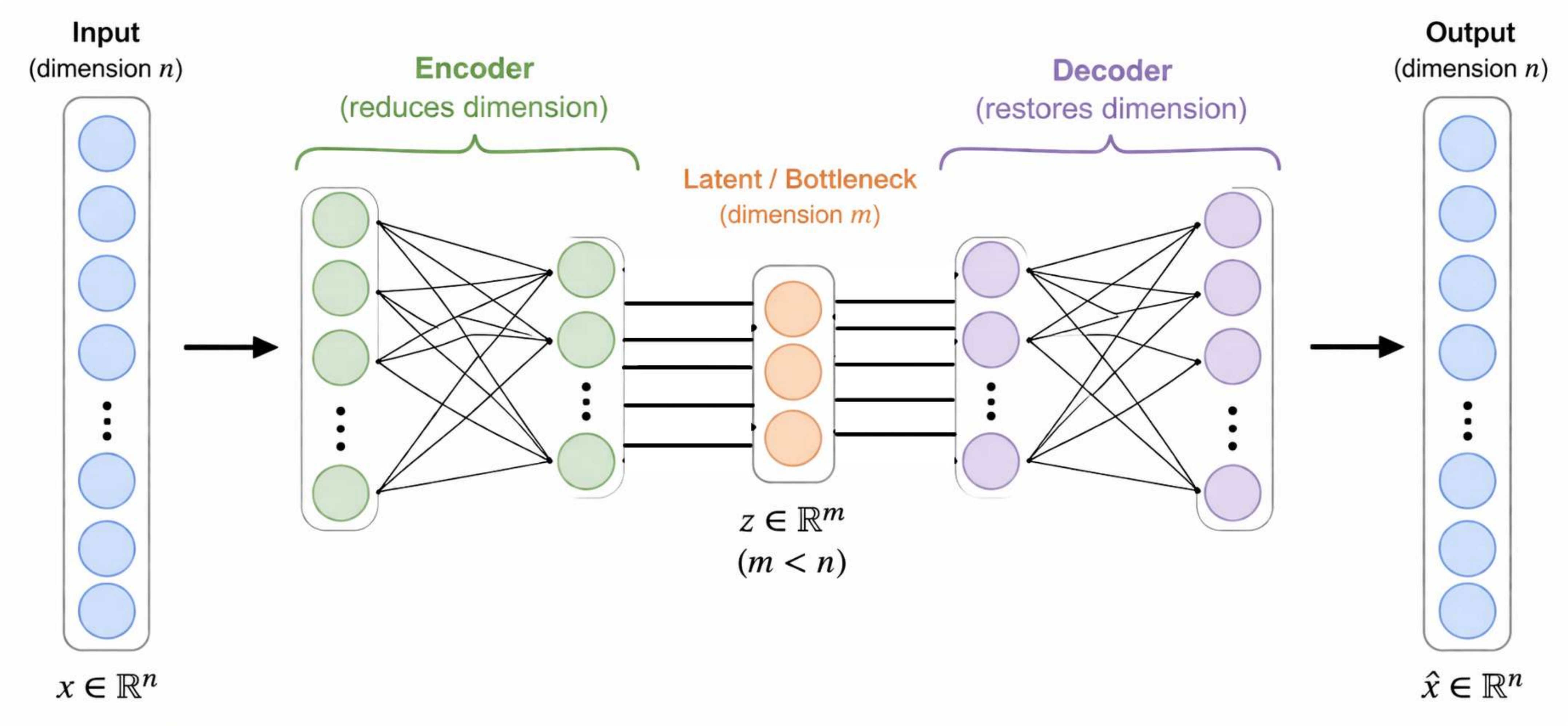

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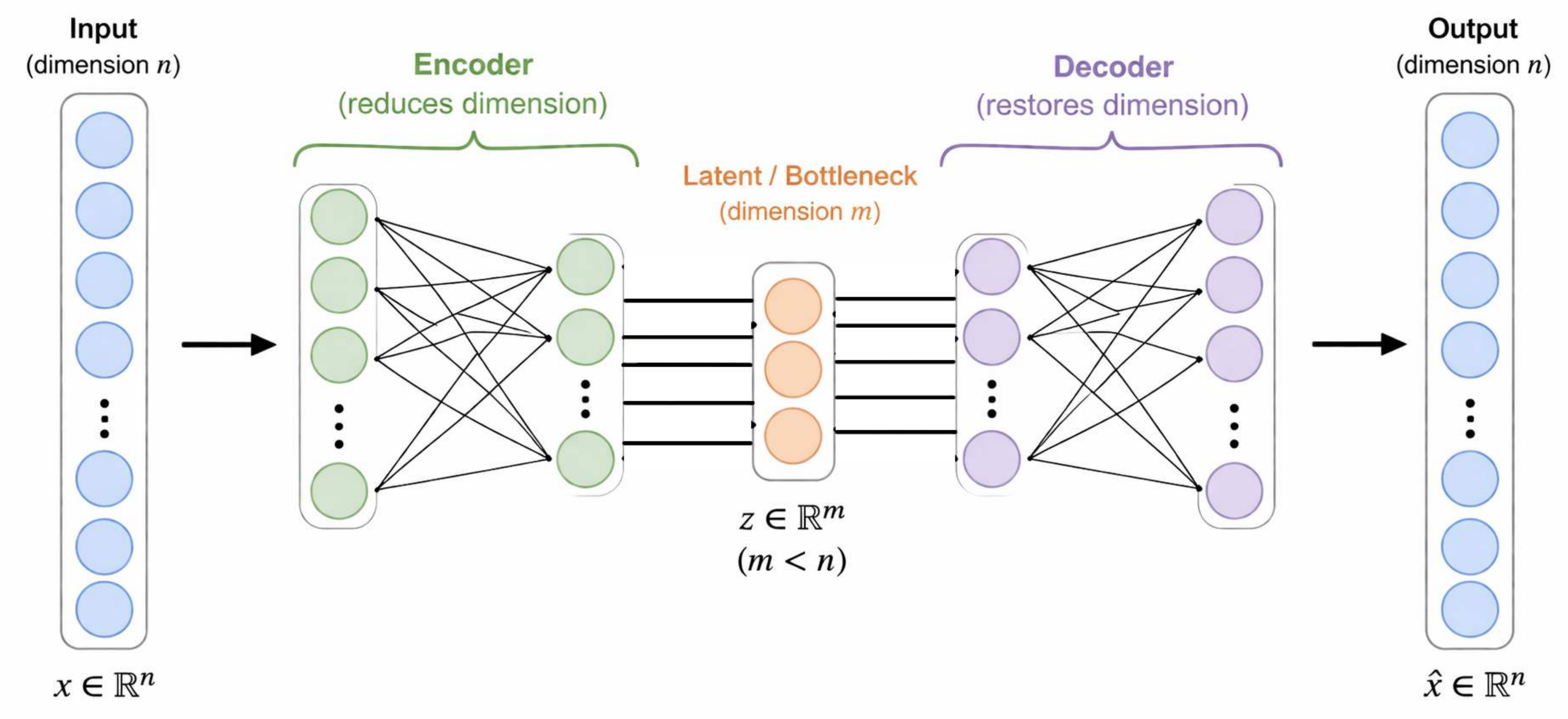

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Architecture and size of Network

Note on neural networks

We have seen neural networks as compositions of smaller systems:

- Layers (weights + biases)
- Residual (skip) connections
- Convolution Layers

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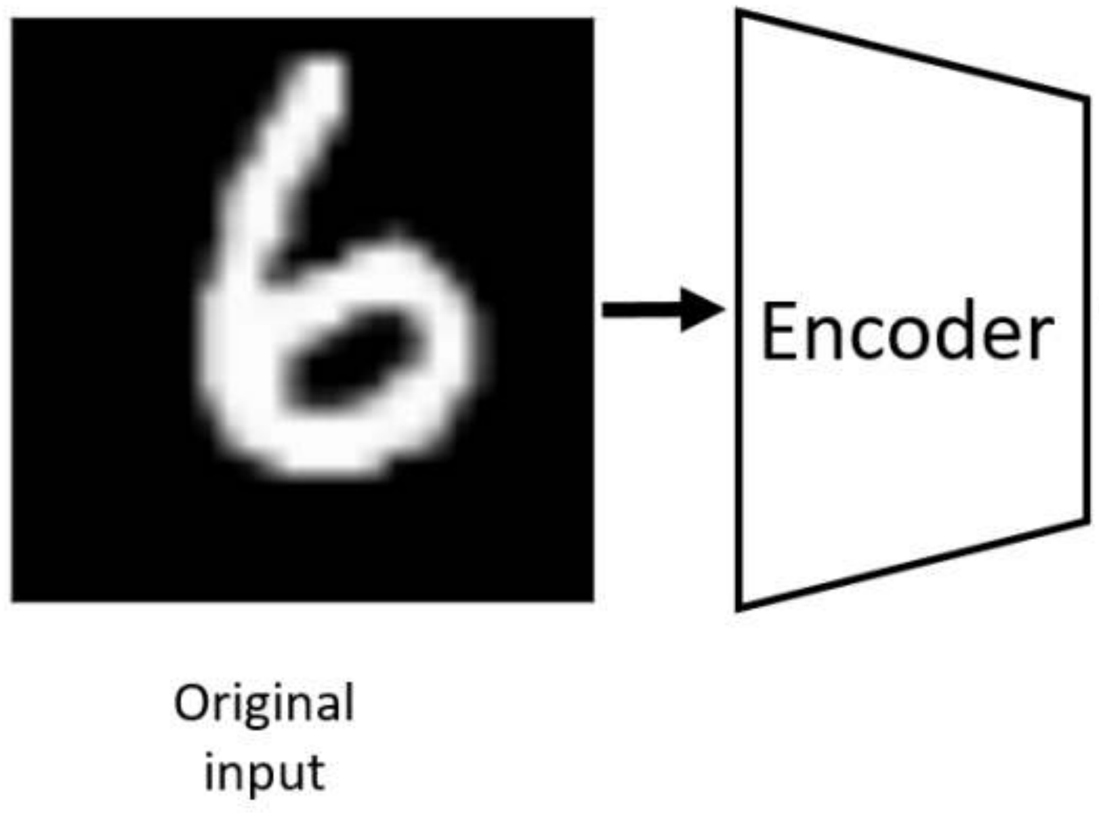
This shows a generalisation:

Neural networks can be built with neural networks as building blocks!

A concrete example

Original
input

A concrete example



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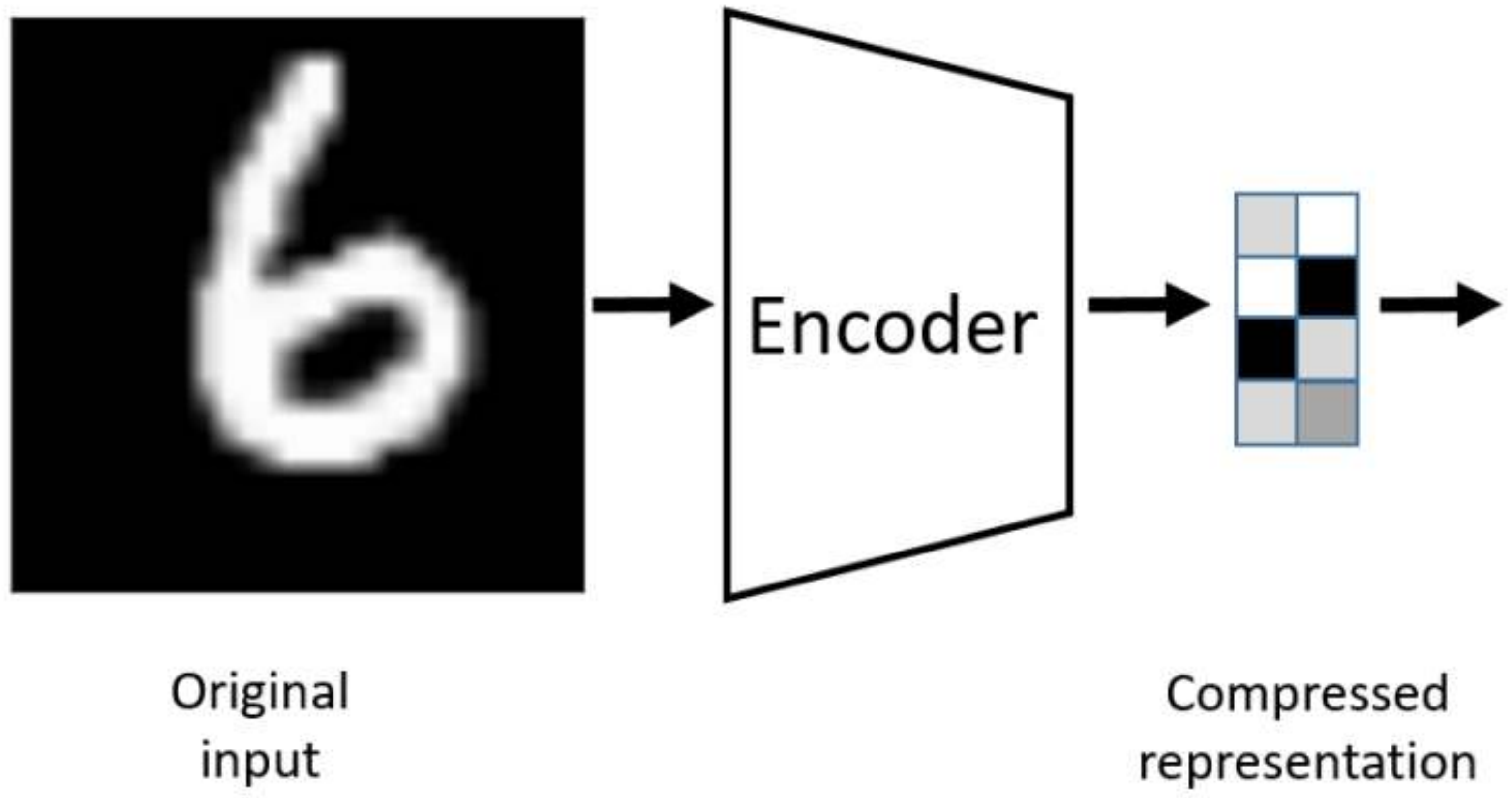

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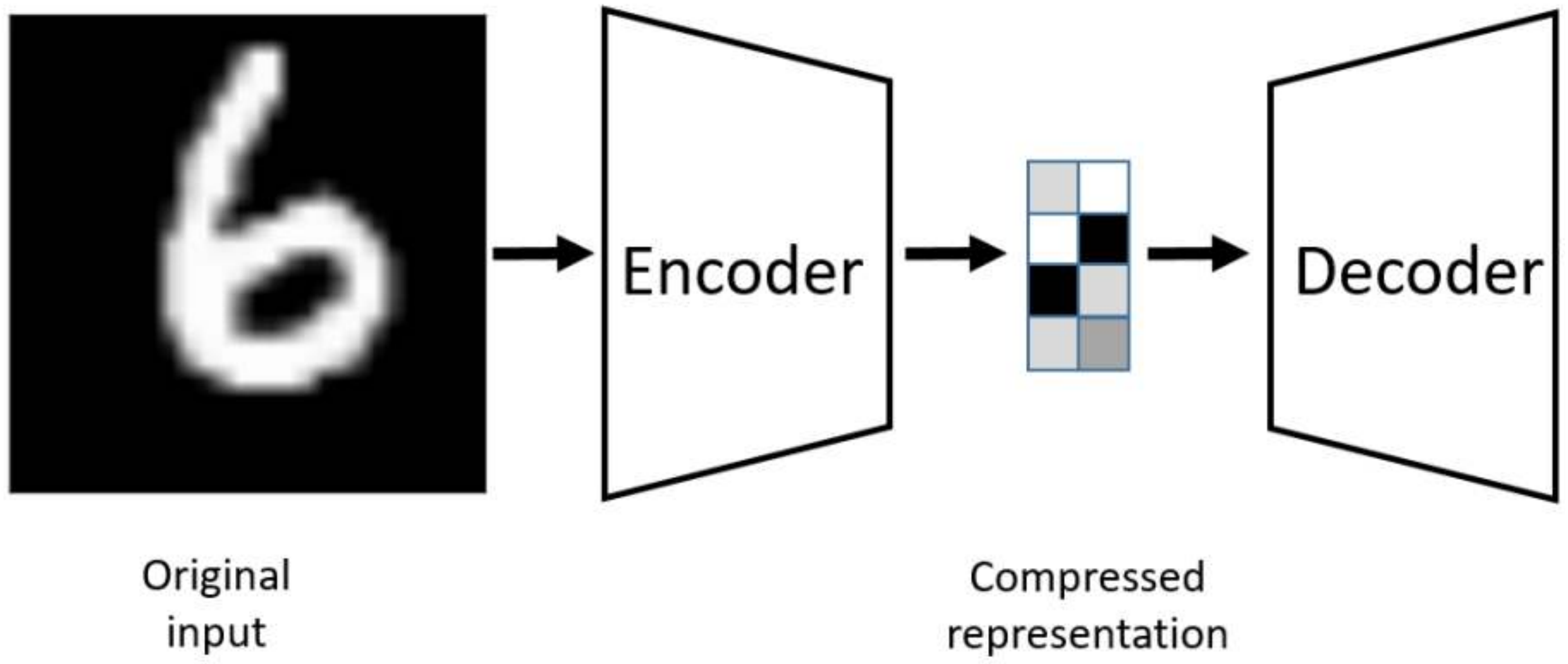

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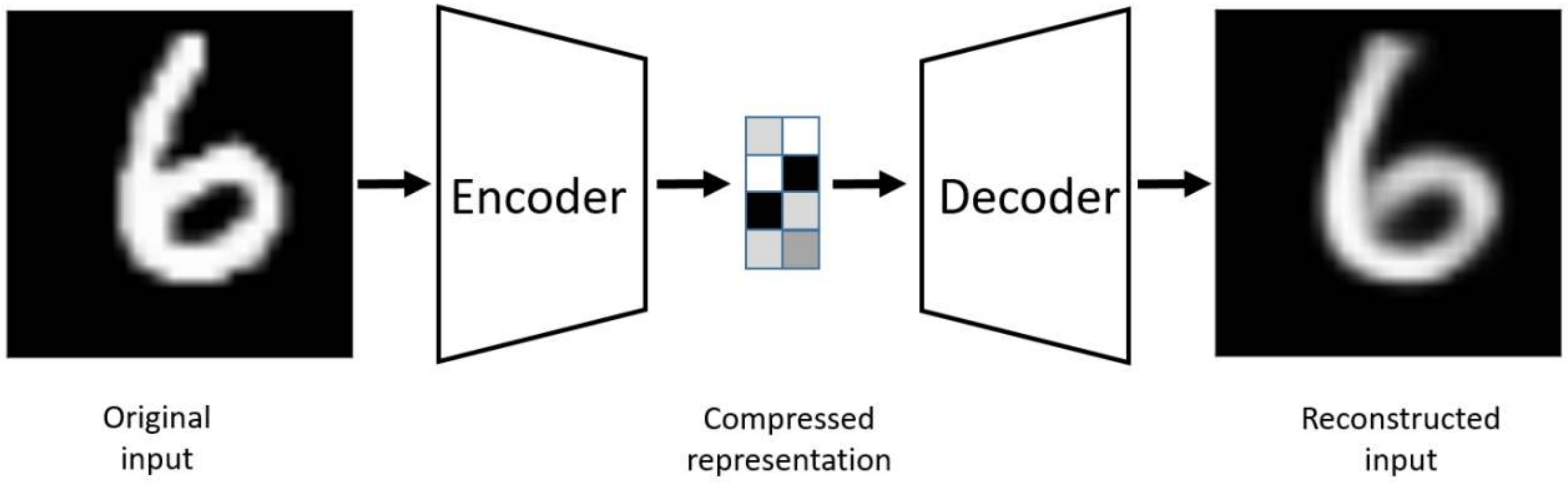

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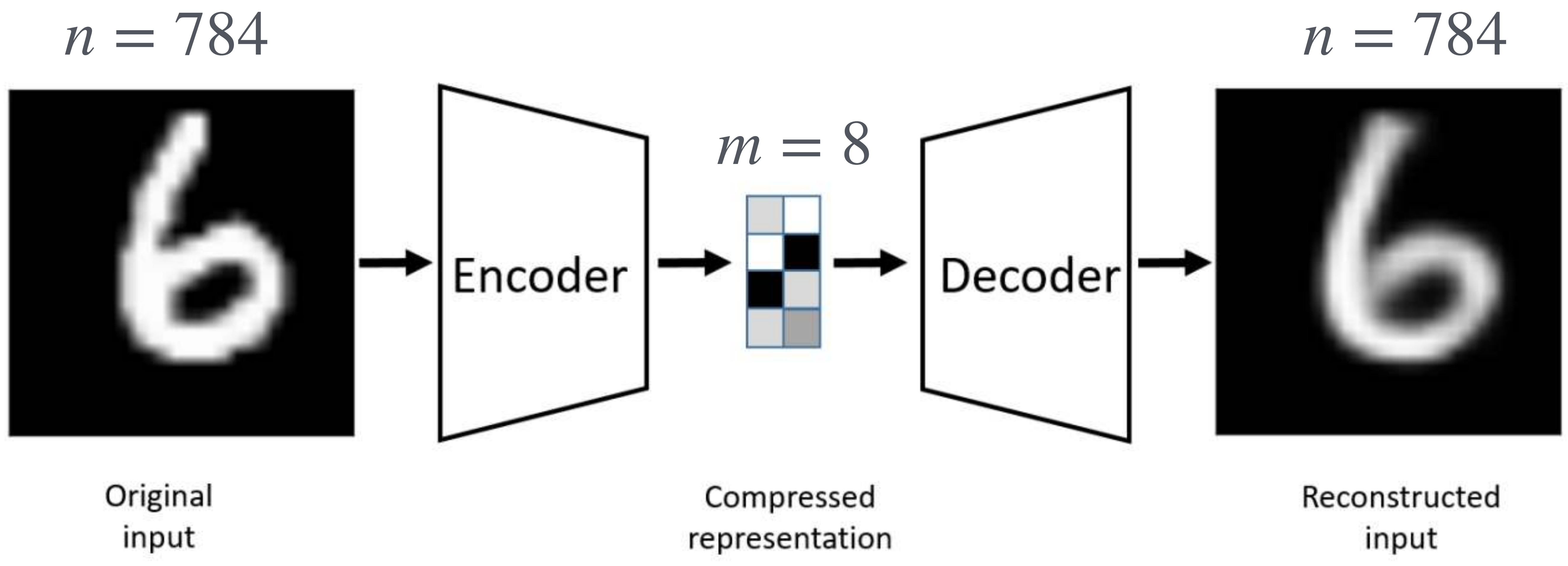

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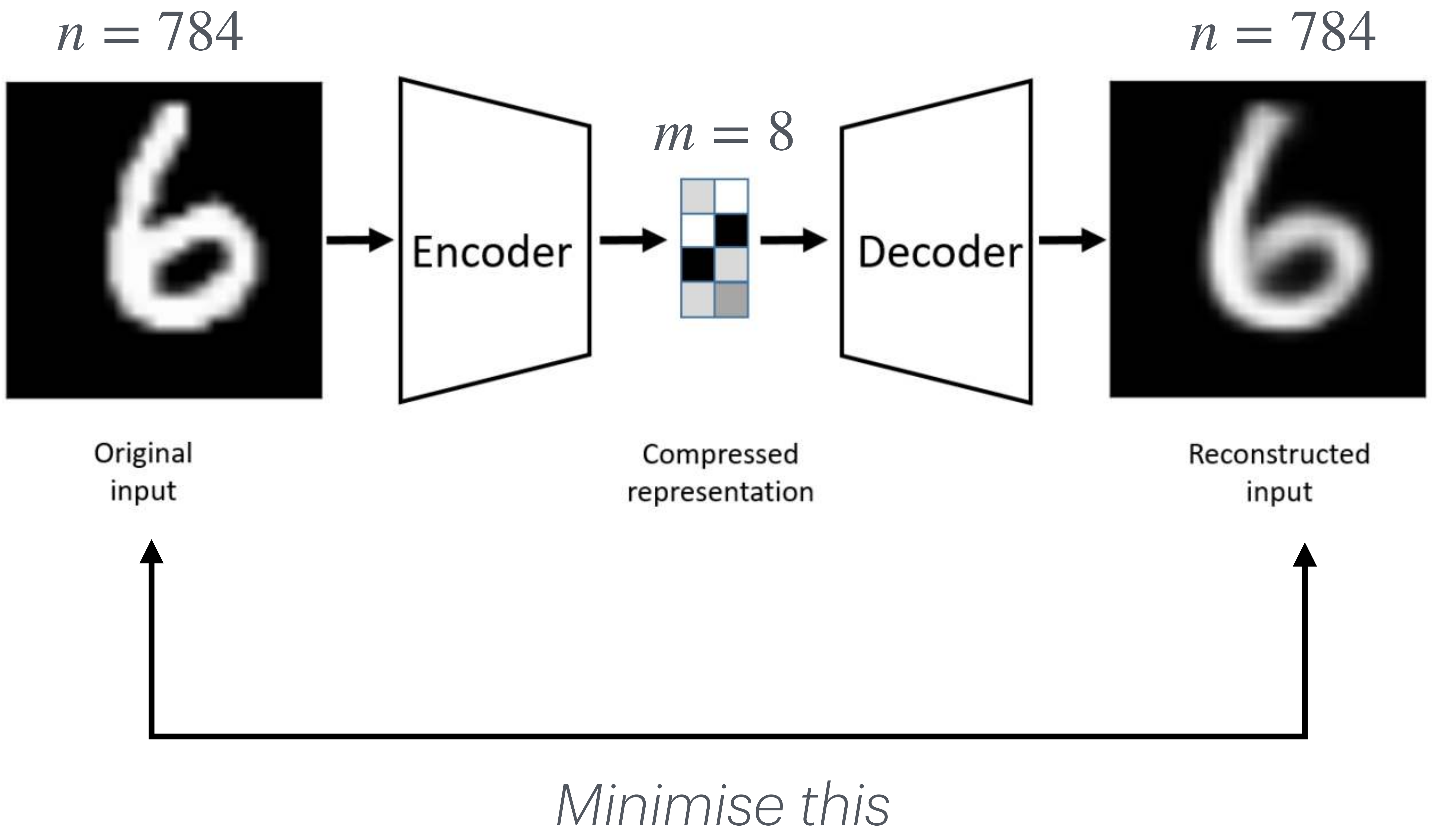

The optimisation problem

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$$\hat{L}(\theta_1, \theta_2) = \frac{1}{n} \sum_{i=1}^{n} \left\| f_{\theta_2}\big(f_{\theta_1}(x_i)\big) - x_i \right\|_2^2$$

The Algorithm

Given dataset X, encoder f_{θ_e}, decoder g_{θ_d}

for each epoch:

    for each mini-batch $B \subset X$:

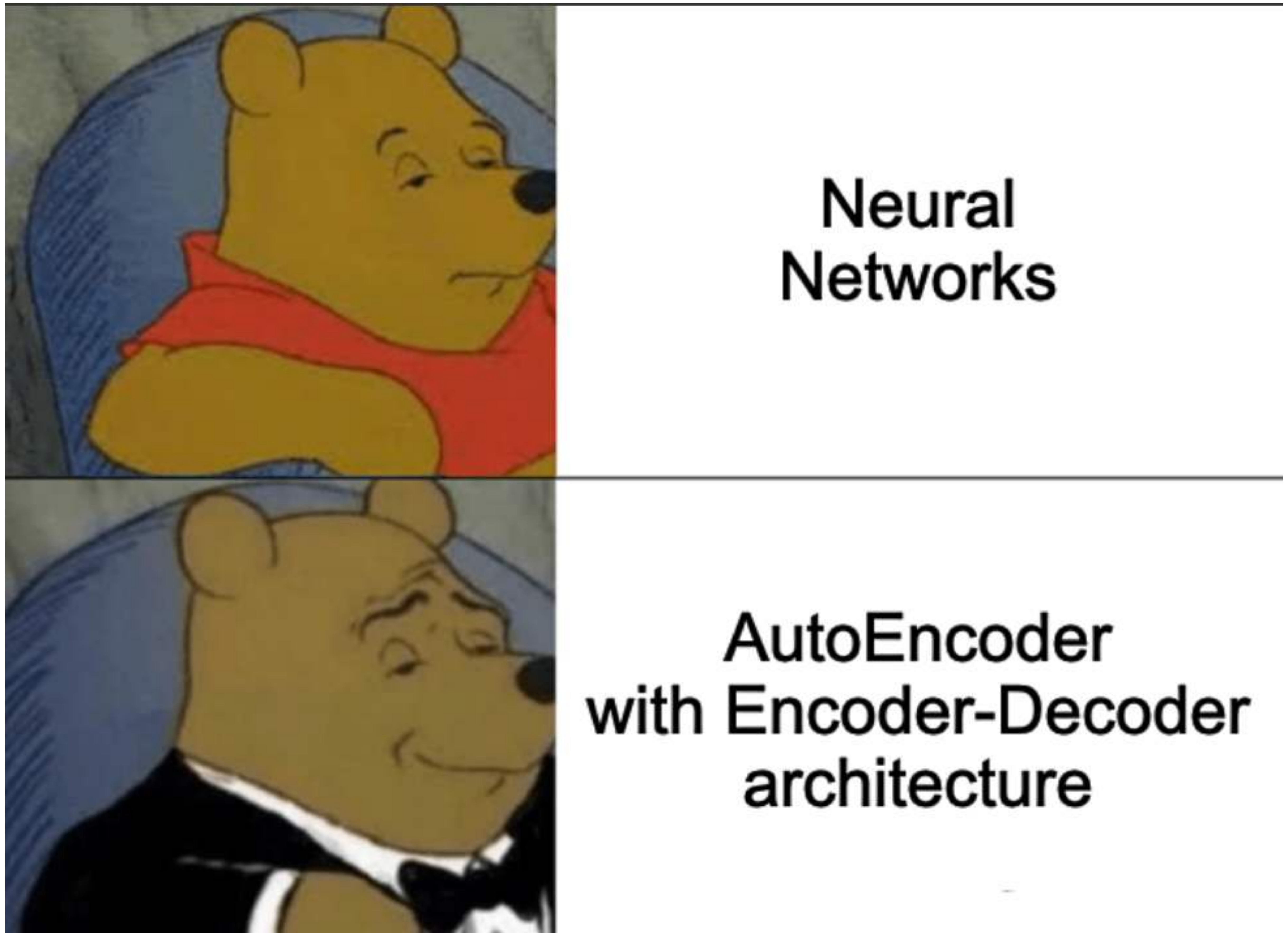
$$z = f_{\theta_e}(x)$$

$$\hat{x} = g_{\theta_d}(z)$$

$$\mathscr{L} = \|x - \hat{x}\|^2$$

$$\theta_e \leftarrow \theta_e - \eta \nabla_{\theta_e} \mathscr{L}$$

$$\theta_d \leftarrow \theta_d - \eta \nabla_{\theta_d} \mathscr{L}$$


Neural Networks
AutoEncoder with Encoder-Decoder architecture

Variational Auto Encoders

The whole secret lies in confusing the enemy, so that he cannot fathom our real intent.
- Sun Tzu, “The Art of War”

Deeper look

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What does the the compression actually do?

Deeper look

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Autoencoders tend to have discrete latent spaces, where each input is mapped to some arbitrary point

So what?

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If we take a random point in the latent space and decode it, we get garbage

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The latent space does not capture the semantic meaning well

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The latent space does not capture the semantic meaning well

Can we make it a nicer space, maybe continuous?

Hunt for a better space

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The encoder predicts a *fixed* output, causing this memorisation

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Can we *vary* the output of the encoder?

Hunt for a better space

The encoder predicts a *fixed* output, causing this memorisation

Can we *vary* the output of the encoder?

Instead of encoding to fixed point, encode using some *randomness*

Randomness

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Randomness should be small enough for the decoder to reconstruct, but large enough for the encoder to not memorise

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But how?

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But how?

Just learn it

Learning the randomness to learn…

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The randomness is parameterised by a Gaussian

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The encoder takes a data point and outputs the mean μ and the log variance $\log \sigma^2$ of the gaussian

Learning the randomness to learn…

The randomness is parameterised by a Gaussian

The encoder takes a data point and outputs the mean μ and the log variance $\log \sigma^2$ of the gaussian

The latent is now a sample from this distribution

An example

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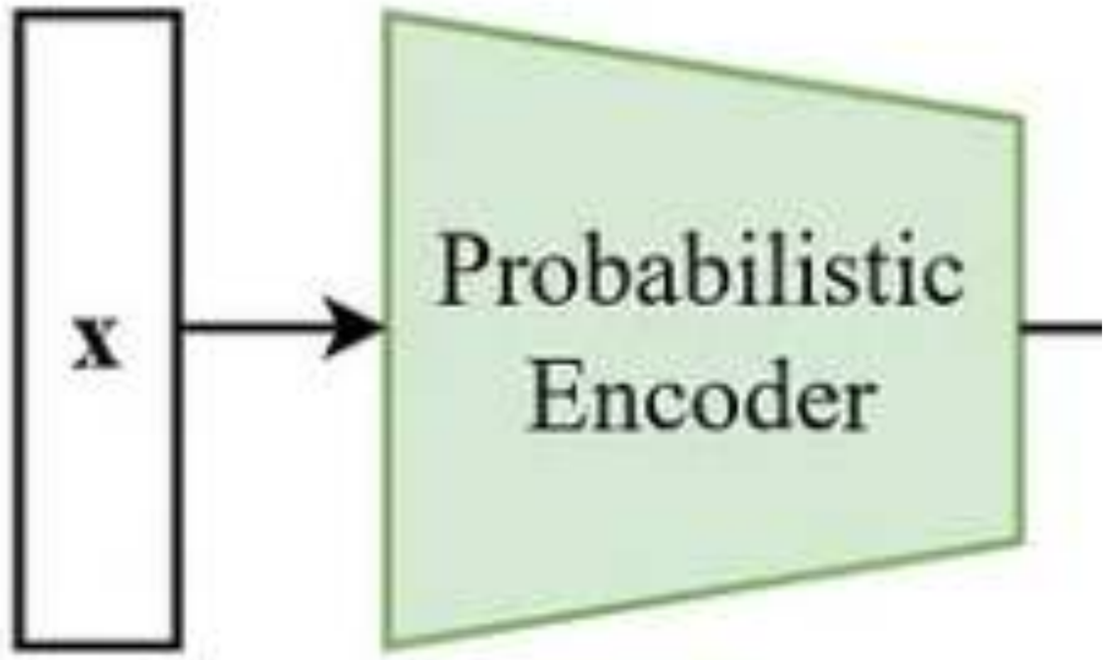

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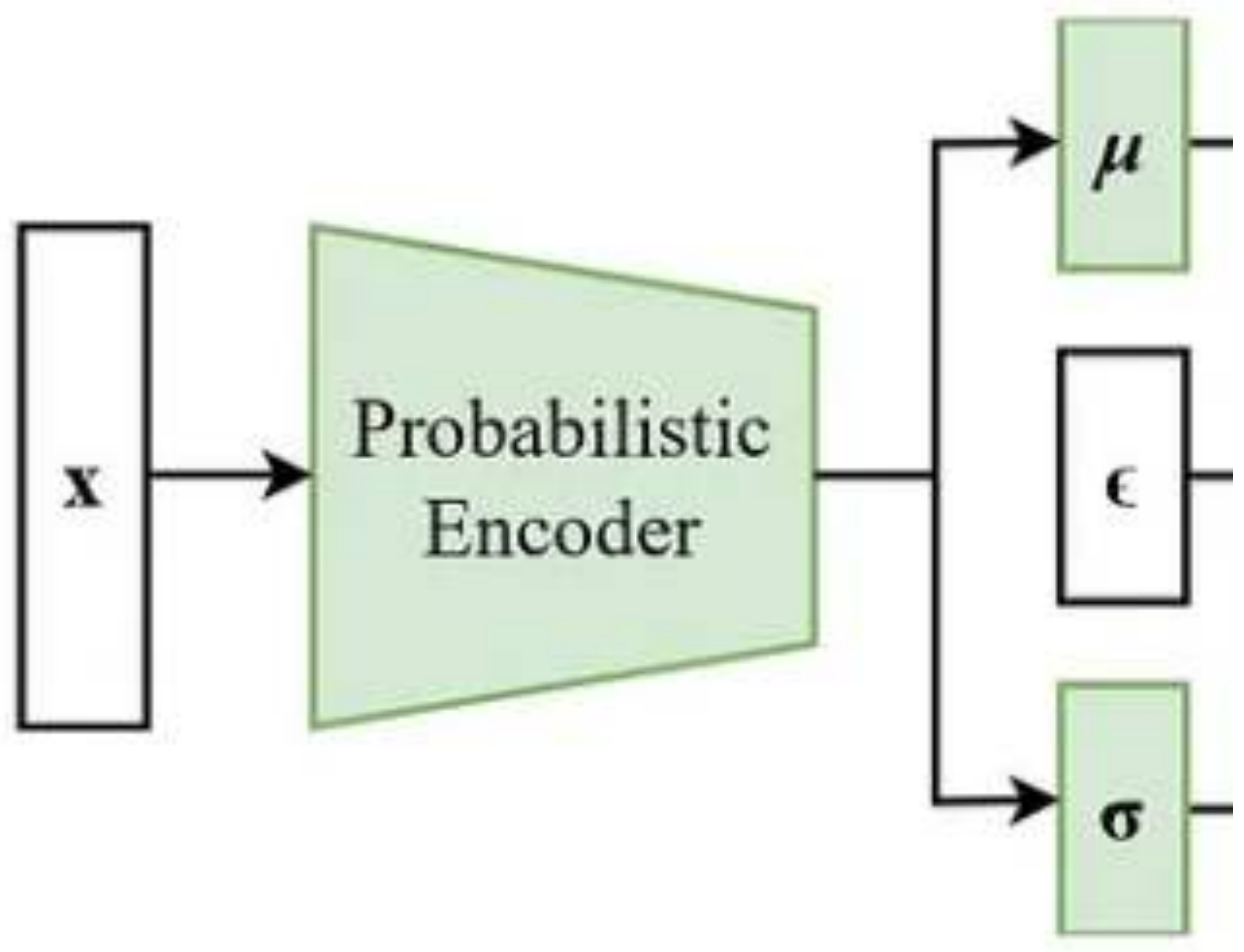

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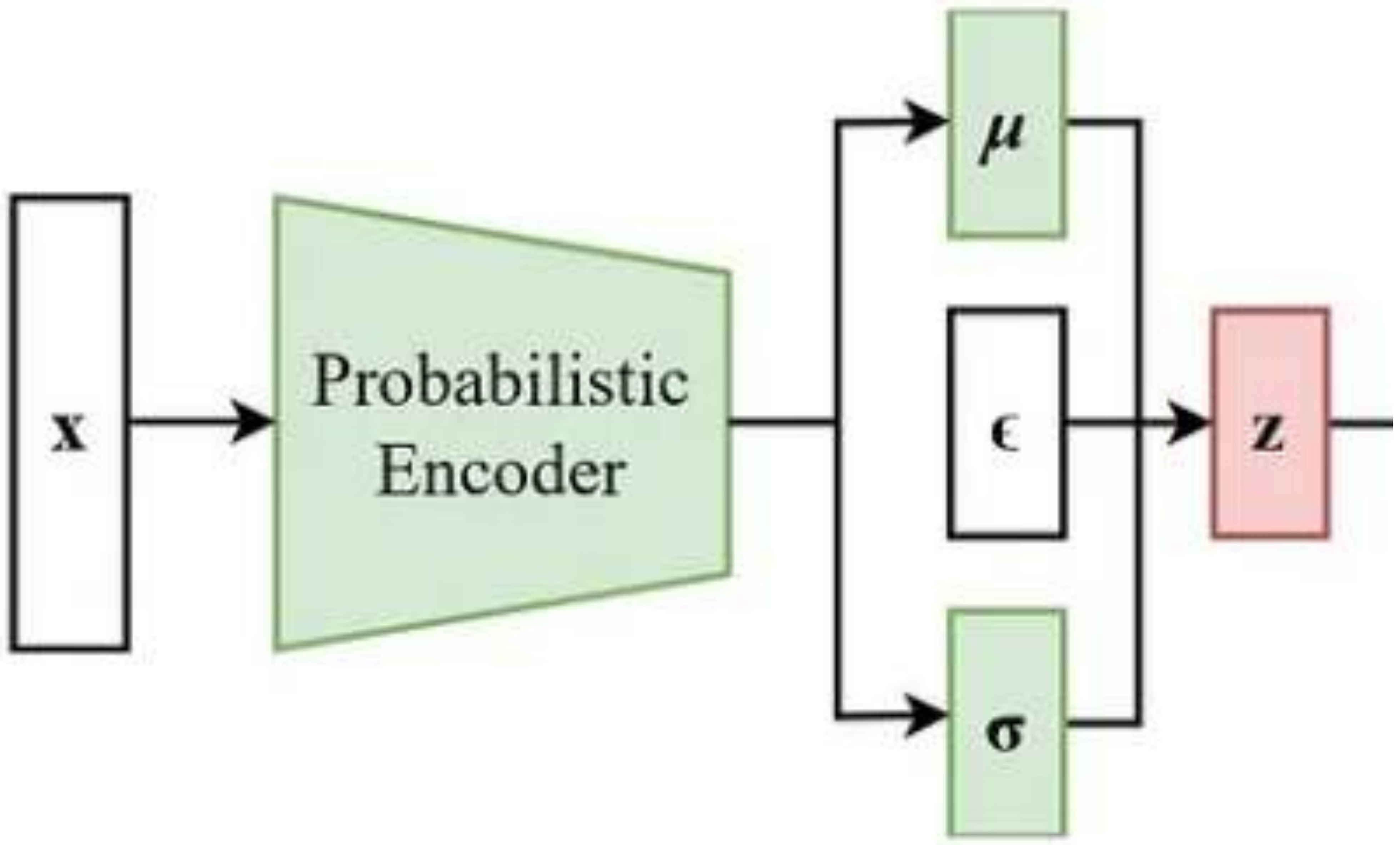

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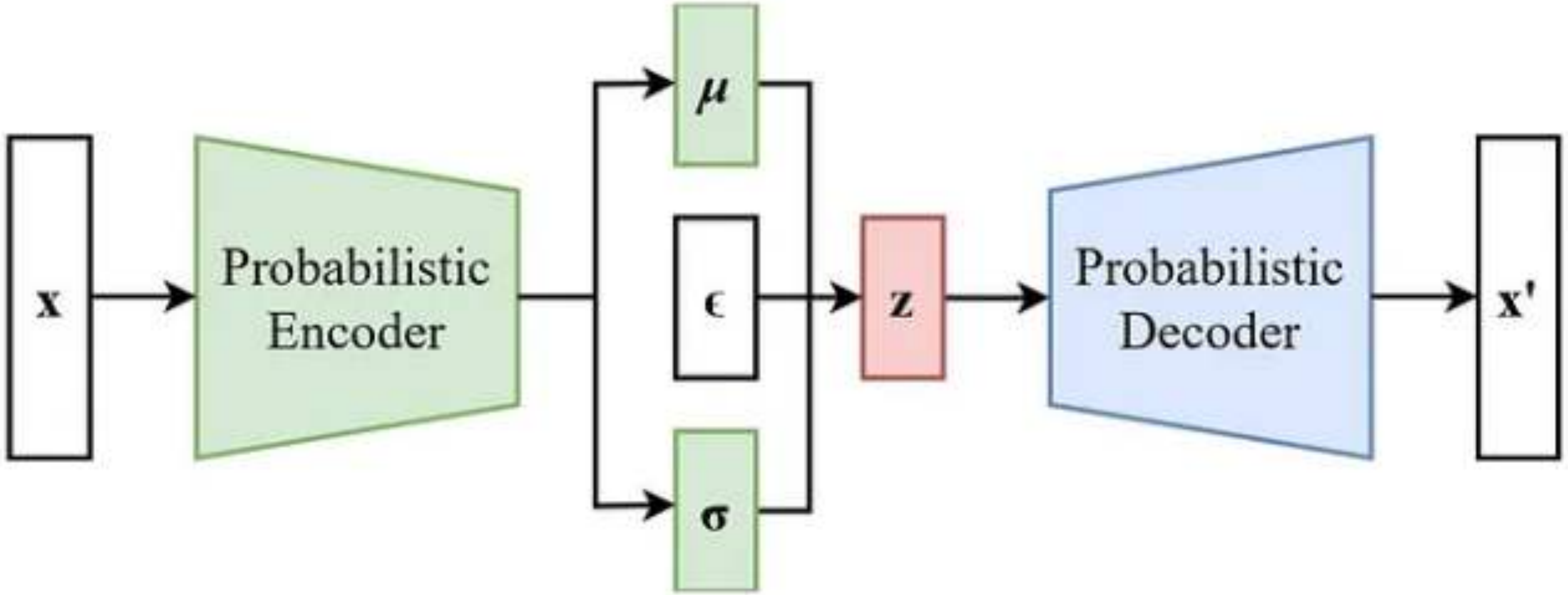

What can do wrong?

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Model always sets the variance to 0, becomes an auto encoder

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Can we penalise the model for being too “discrete”?

The penalty

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Force the encoder to produce outputs close to a standard Gaussian by adding a KL loss

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Force the encoder to produce outputs close to a standard Gaussian by adding a KL loss

$$\mathrm{KL}\left(p \| q\right) = \int p(x) \log \frac{p(x)}{q(x)} dx$$

Measures how close two distributions are

The closed form

$$p = \mathcal{N}(\mu_p, \sigma_p^2 I), \quad q = \mathcal{N}(\mu_q, \sigma_q^2 I)$$

The closed form

$$\mathrm{KL}\left(p\|q\right) = \frac{1}{2}\left[d\frac{\sigma_p^2}{\sigma_q^2} + \frac{\|\mu_p - \mu_q\|^2}{\sigma_q^2} - d + d\log\left(\frac{\sigma_q^2}{\sigma_p^2}\right)\right]$$

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Finally…

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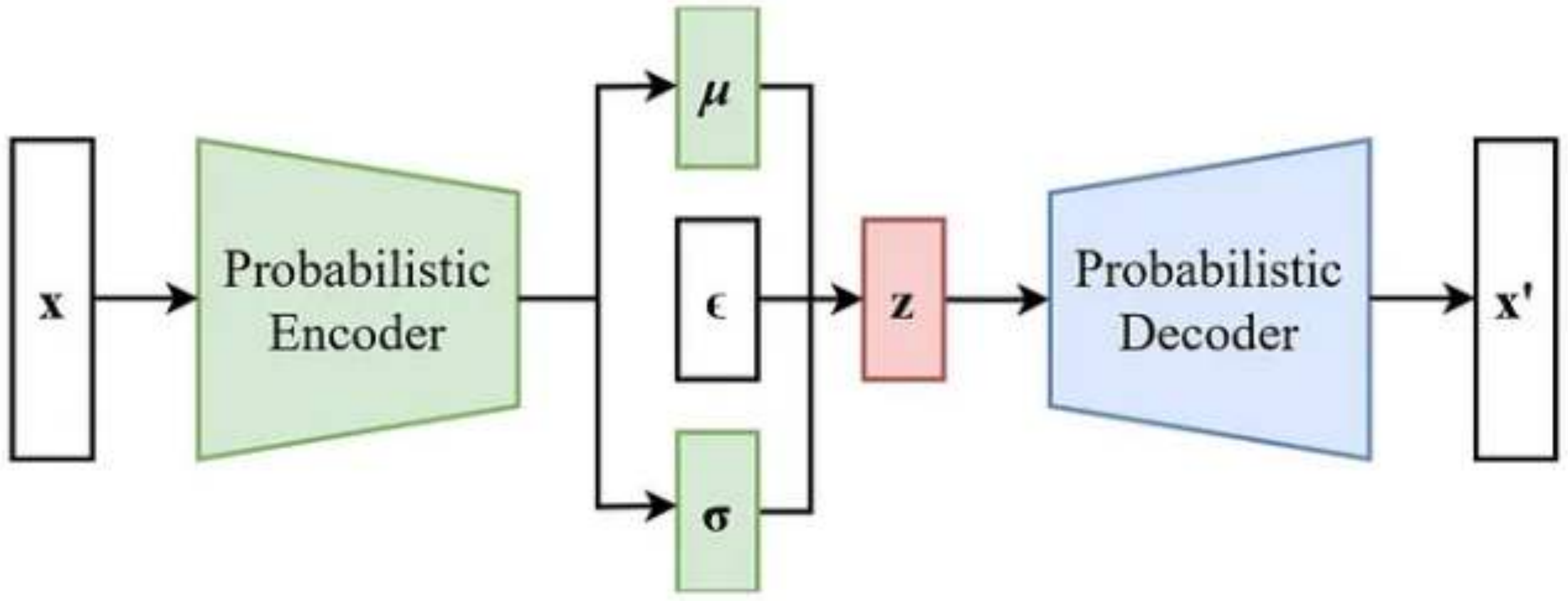

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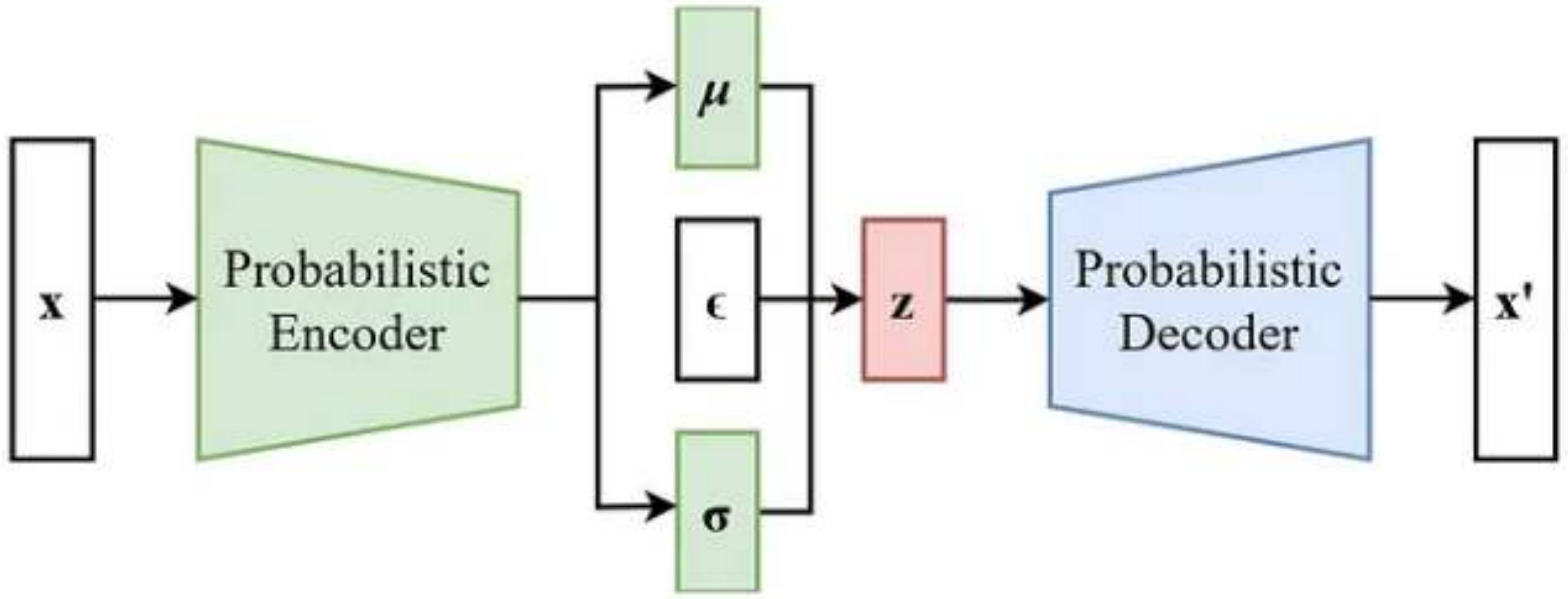
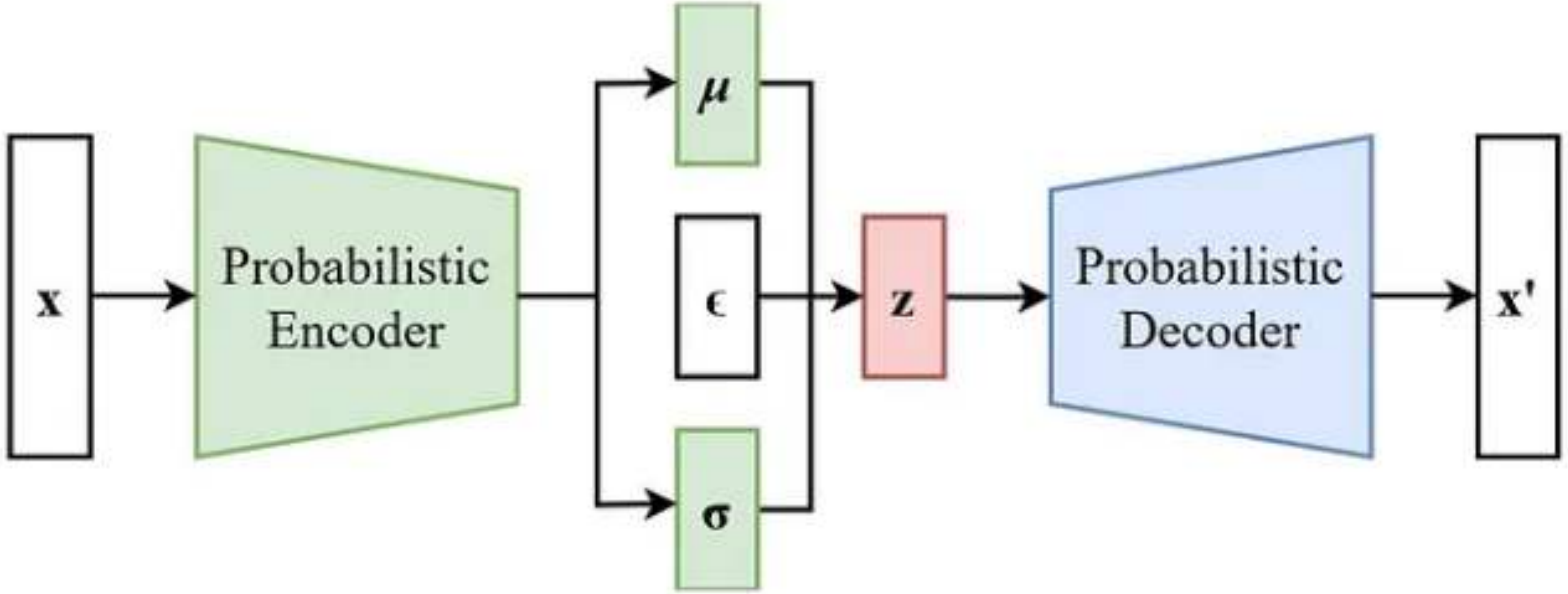


$$L(\theta_1, \theta_2) = \|x - f_{\theta_2}(z)\|_2^2 + \beta \cdot D_{KL}(q_{\theta_1}(z \mid x) \parallel \mathcal{N}(0,I))$$

Finally...



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Recon. Loss

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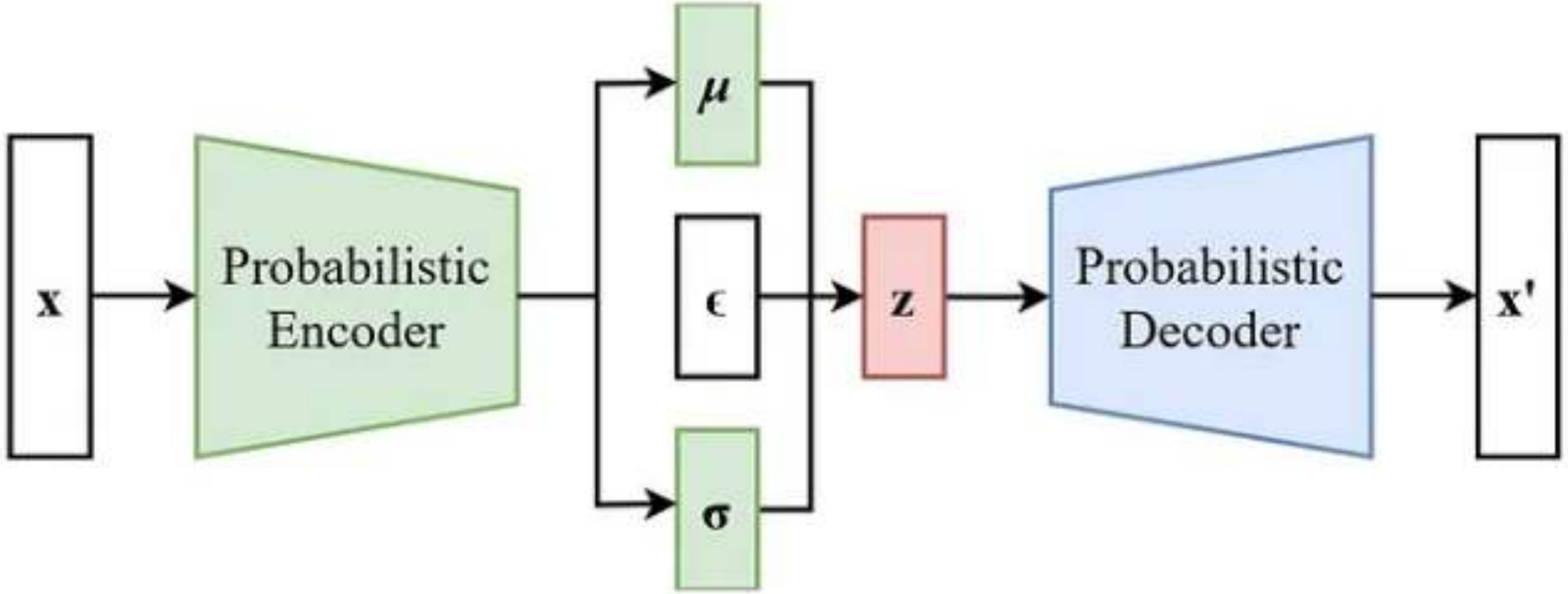

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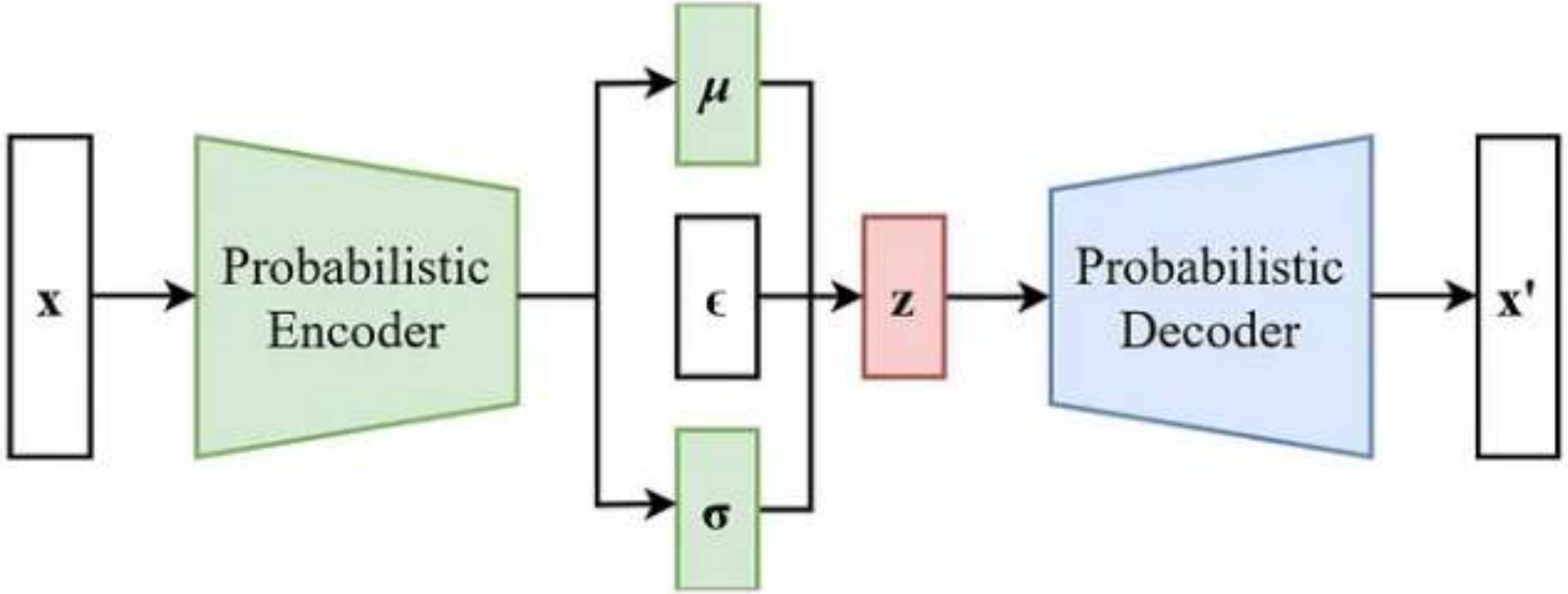

$$L(\theta_1, \theta_2) = \underbrace{\boxed{\|x - f_{\theta_2}(z)\|_2^2}}_{\text{Recon. Loss}} + \overbrace{\boxed{\beta}}^{\text{Regularization Strength}} \cdot \underbrace{\boxed{D_{KL}(q_{\theta_1}(z \,|\, x) \;\|\; \mathcal{N}(0,I))}}_{\text{KL Penalty}}$$

SOTA

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Stable Diffusion XL's VAE reduces the dimension by 48 times

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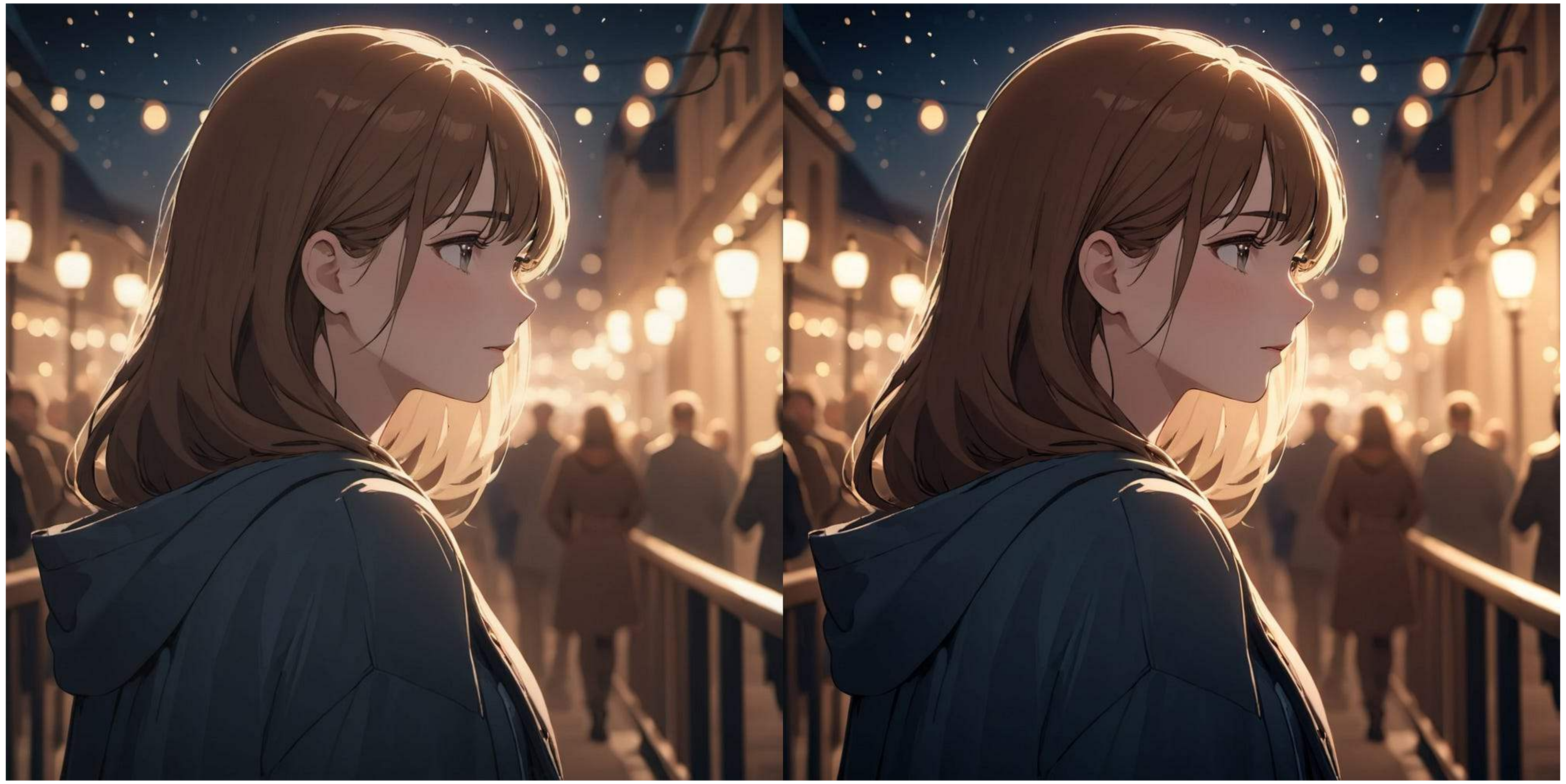

Original Reconstructed

SOTA

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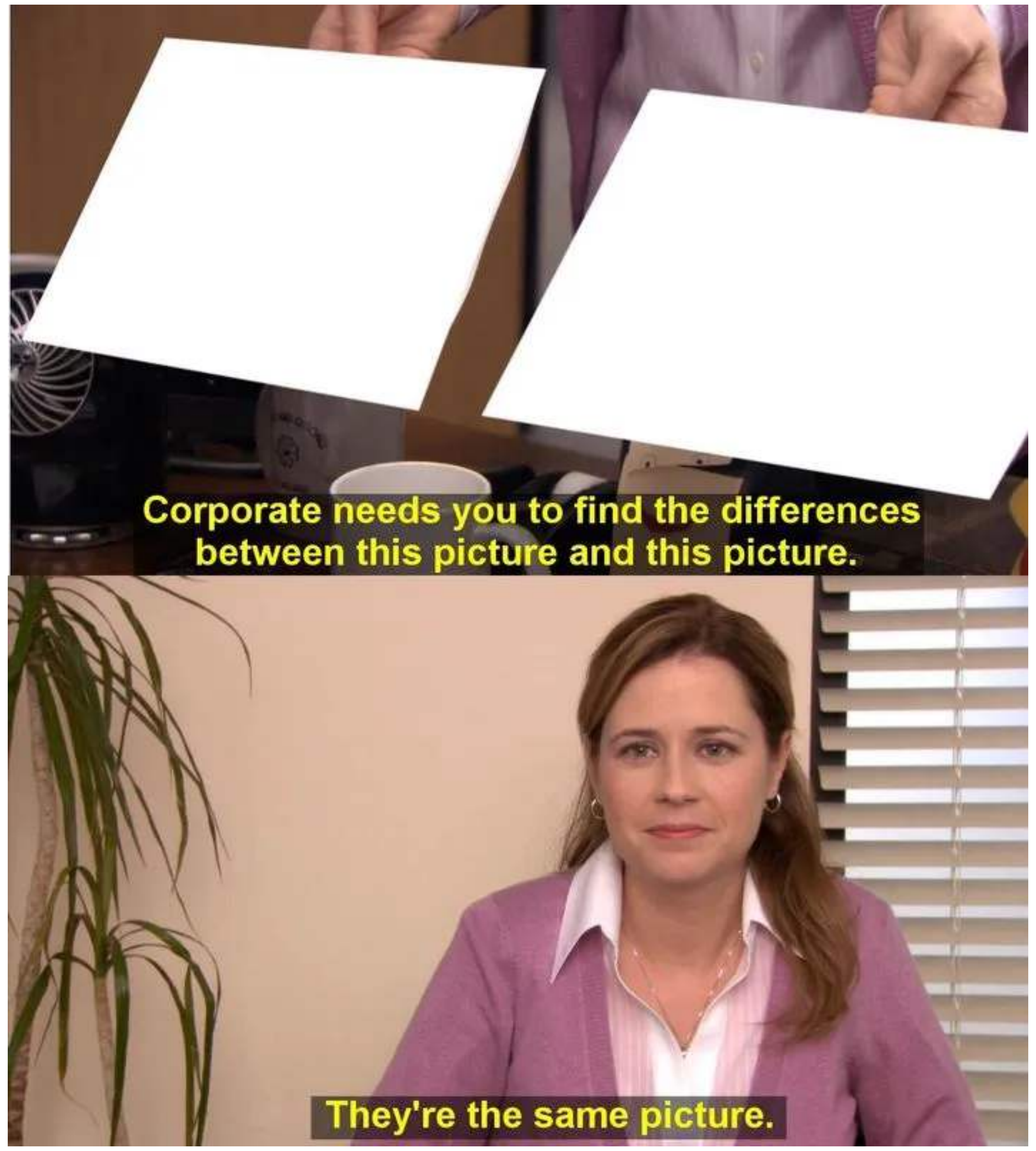
Original

Reconstructed



Sequence Modelling

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Inputs are sequences $\{x_t\}$ and we wish to model them

Given$\{x_1, \cdots, x_{t-1}\}$ can we predict x_t?

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Examples:

- Stock prediction
- Next token prediction (language models!)
- Weather Prediction, etc

Sequence Modelling - Challenges

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Variable length inputs

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What if sequences are very long?

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Remembering “context”

Sequence Modelling - Challenges

Variable length inputs

What if sequences are very long?

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Remembering “context”

Predictions need to somehow use the entire history

Approach #1: RNNs

Key Idea

Represent the entire context as a hidden variable $\boldsymbol{h} \in \mathbb{R}^d$

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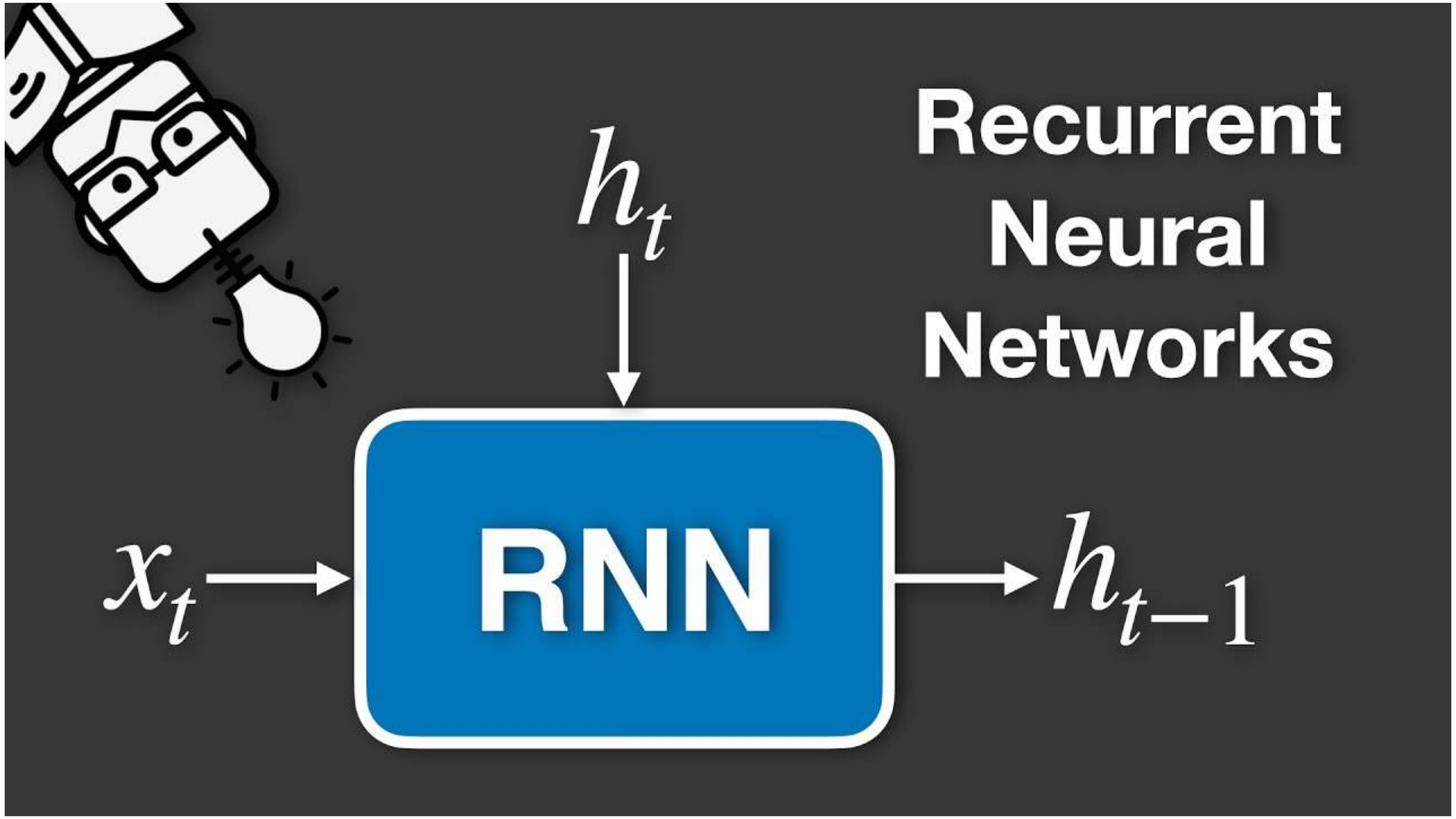
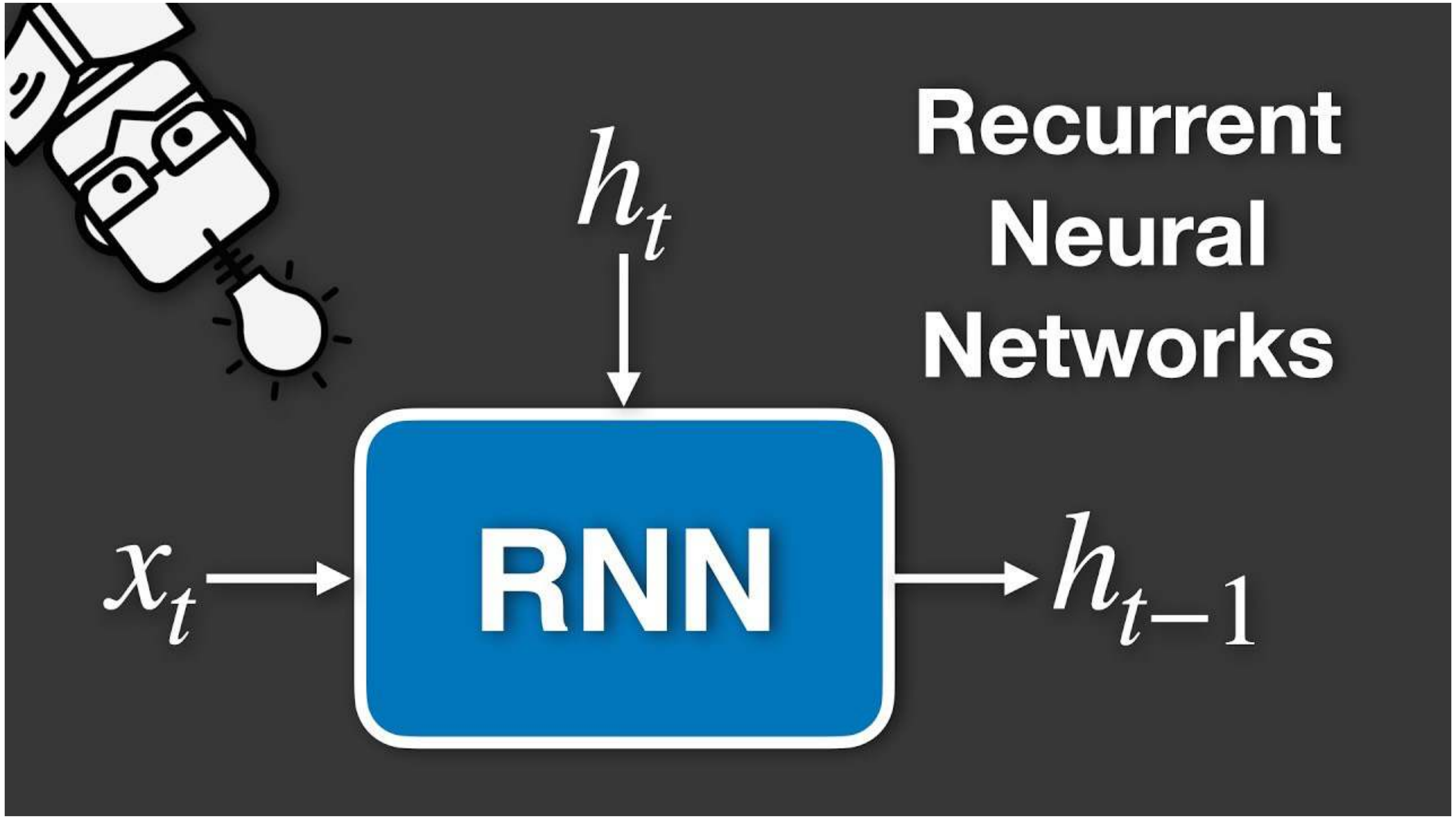
Represent the entire context as a hidden variable $\boldsymbol{h} \in \mathbb{R}^d$

Given the hidden variable, generate the next output

Recurrence?



Recurrence?



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Hard to parallelise - only run sequentially

Increases cost of training and inference

What now?

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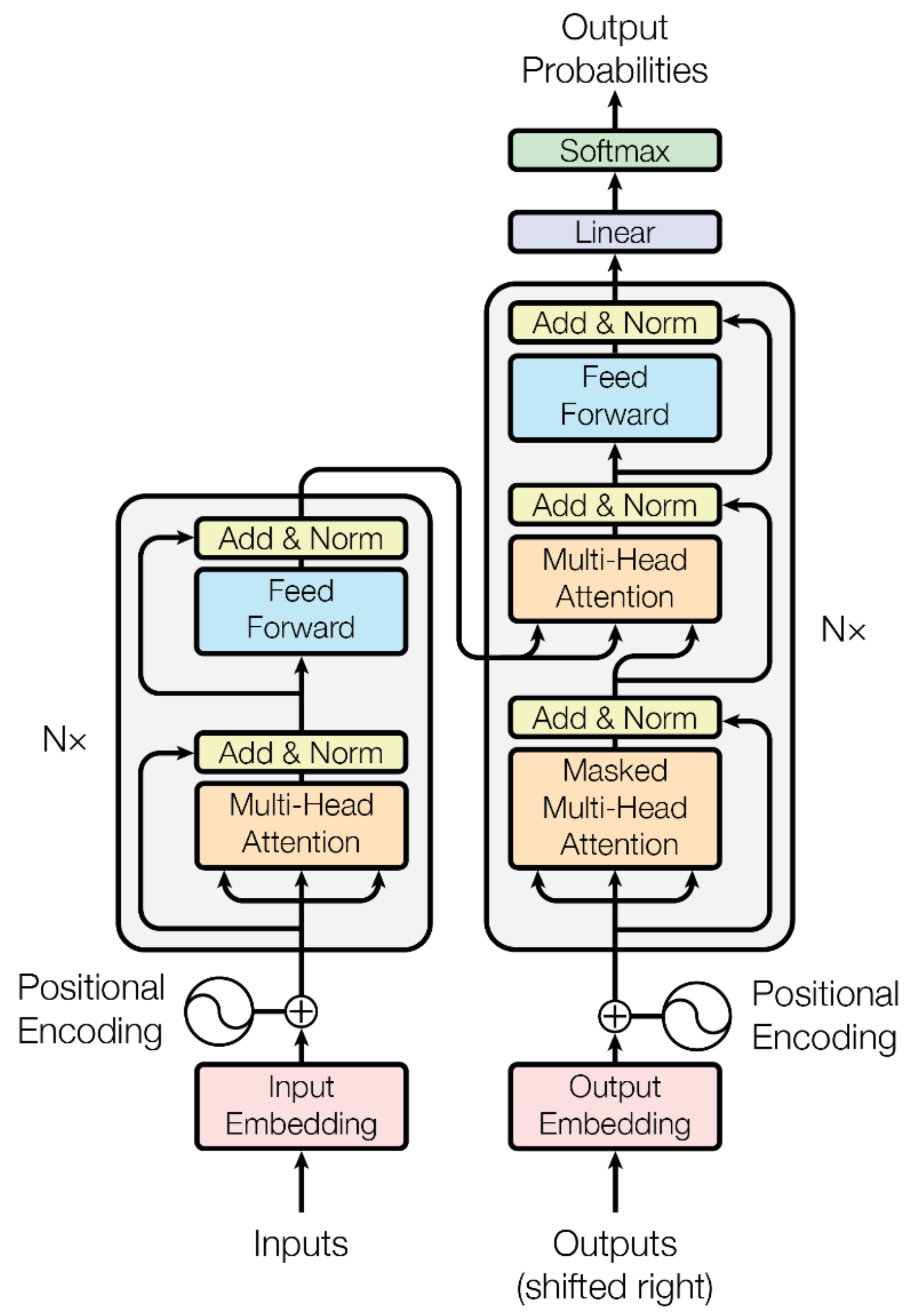
Attention Is All You Need



What now?

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